Section 2.3
Bipolar junction transistors - BJTs

Single junction devices, such as p-n and Schottkty diodes can be used to obtain rectifying I-V characteristics, and to form electronic switching circuits

The *transferred-resistance or transistor* is a multijunction device that is capable of

- Current gain
- Voltage gain
- Signal-power gain

The transistor is therefore referred to as an active device, where-as the diode is passive
Basic action - control of I at one terminal by a voltage applied across two other terminals

There are three major types of transistor

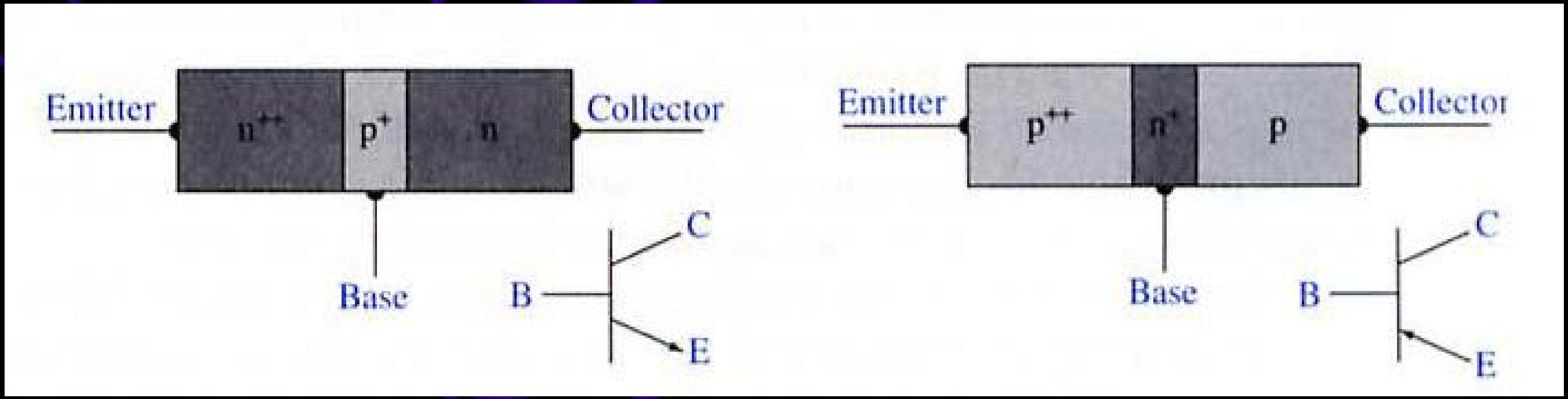
- Bipolar junction transistor or BJT
- Metal-oxide field effect transistor or MOSFET
- Junction FET or JFET

In this case we will consider the first two, as they account for the vast majority of silicon devices produced today

The BJT has three separately doped regions and two p-n junctions, which are sufficiently close together that they interact with each other. In this sense a BJT must NOT be considered as two back-to-back diodes.

The BJT is a voltage-controlled current source. We will consider the various factors that determine the current gain, and the limitations imposed by real device characteristics.

BJT structure - three regions, two p-n junctions, three terminals (emitter, base, collector)

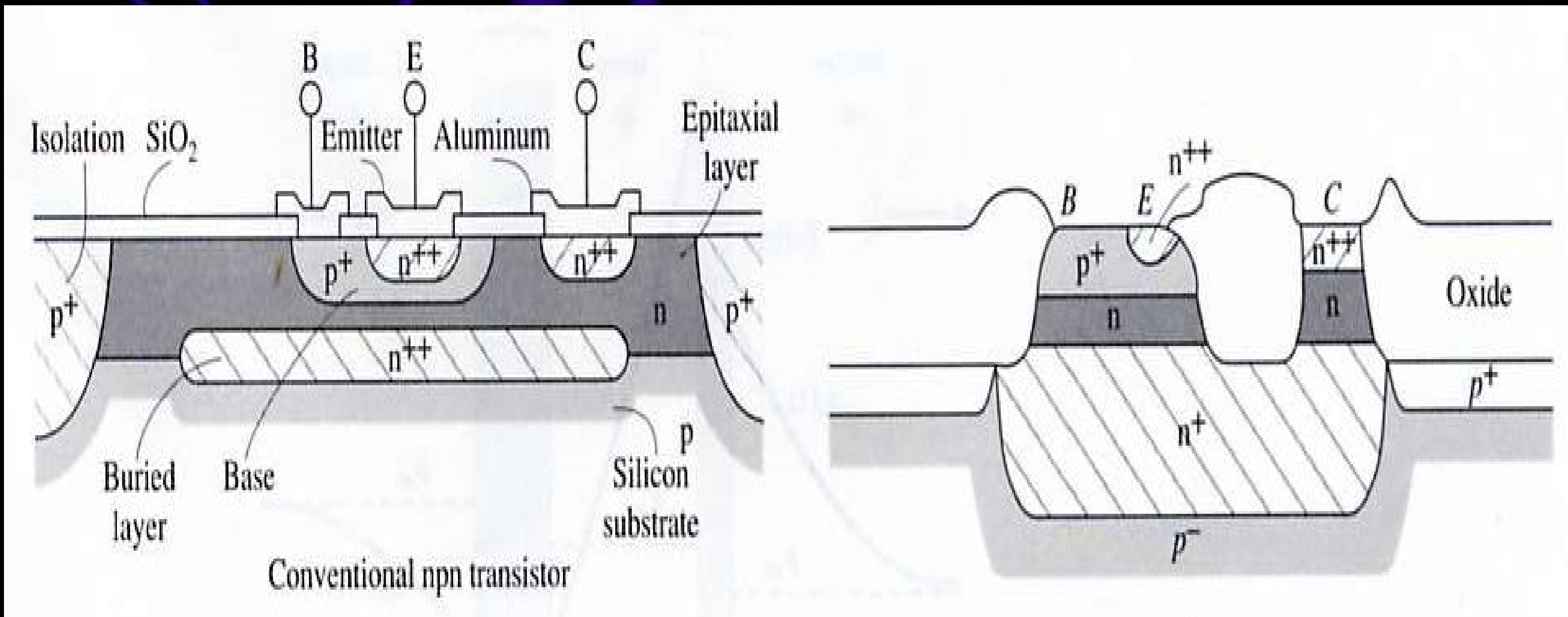


Devices can be p-n-p, or n-p-n structures

- Width of the base is small compared to the minority carrier length
- Emitter is normally heavily doped, the collector has light doping

The concepts you learnt earlier for p-n junctions are going to be built upon here - if you are not happy with these you must revise them now

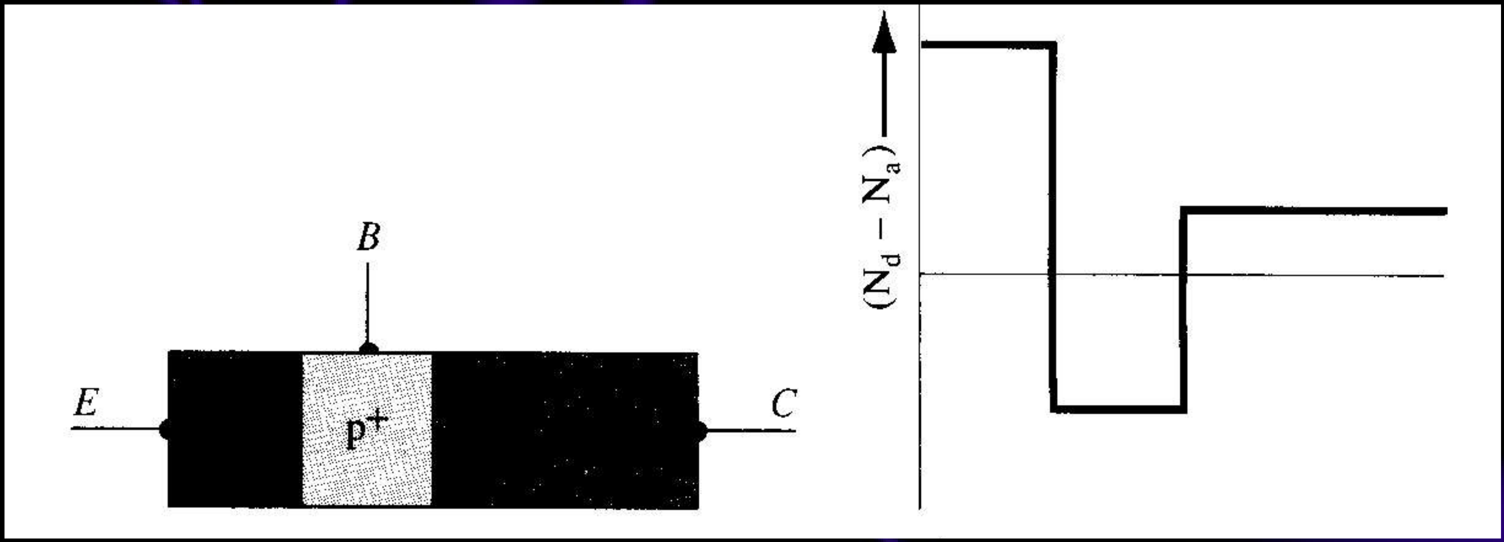
These schematic representations are useful for analysing the basic operation of the structure. However, real devices are not symmetrical, and we will consider the impact of this on device operation later



Real devices look more like this; conventional device (left), oxide-isolated device (right)

Principle of operation

- Consider an npn device - pnp devices behave similarly
- Assume an idealised structure such as that below - uniform and even doping



First let us consider operation in the *forward-active* mode: B-E junction is forward biased so electrons can be injected from the emitter to the base, B-C junction is reverse biased.

Under these conditions we can expect

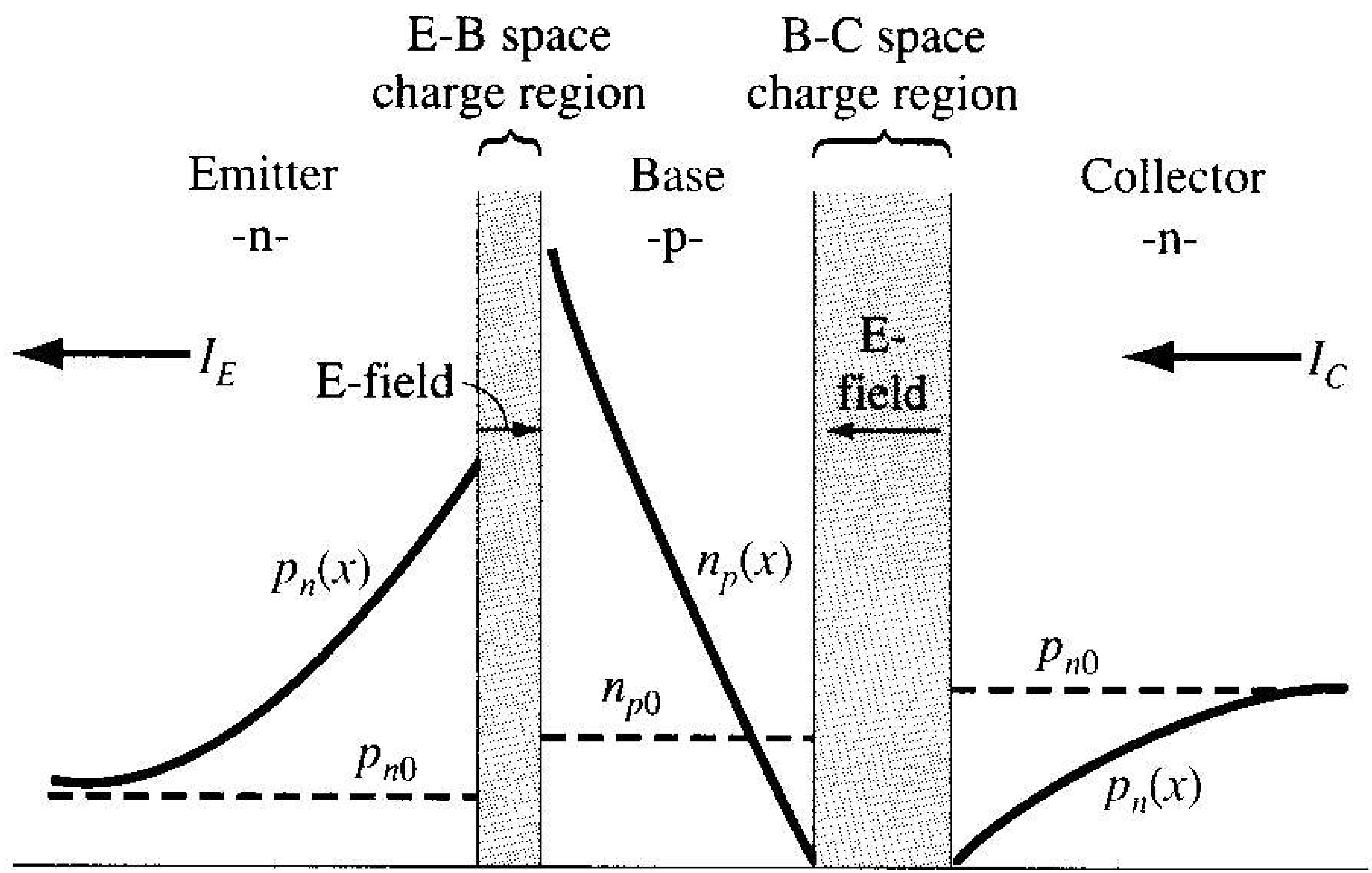
- Large concentration of minority carriers to appear in the base near the B-E junction, due to injection of electrons from the emitter

- A concentration gradient will be created within the base region in terms of minority carriers; this will generate a diffusion current through the base

- The B-C junction is reverse biased, so majority carriers (holes) from the base do not enter the collector in large concentrations; however, the minority carriers that diffuse to the B-C junction will be readily swept across

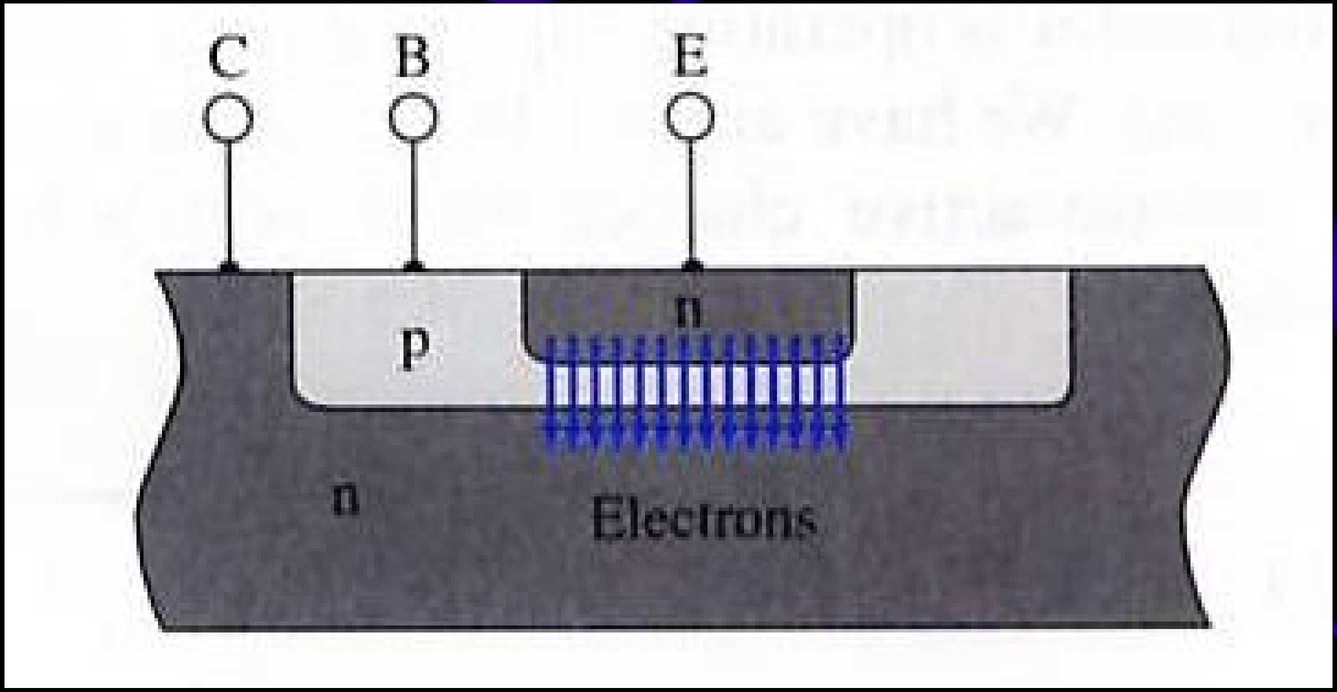
- If the base region is very narrow, comparable to electron diffusion lengths, few electrons will be lost through recombination and a collector current will be measured

The MINORITY carrier concentrations will look like -



The number of electrons per unit time reaching the collector is proportional to the number of electrons injected into the base. The number of injected electrons is a function of the B-E voltage.

To a first approx. the collector current is independent of the reverse biased B-C voltage - thus, the device looks like a constant current source.



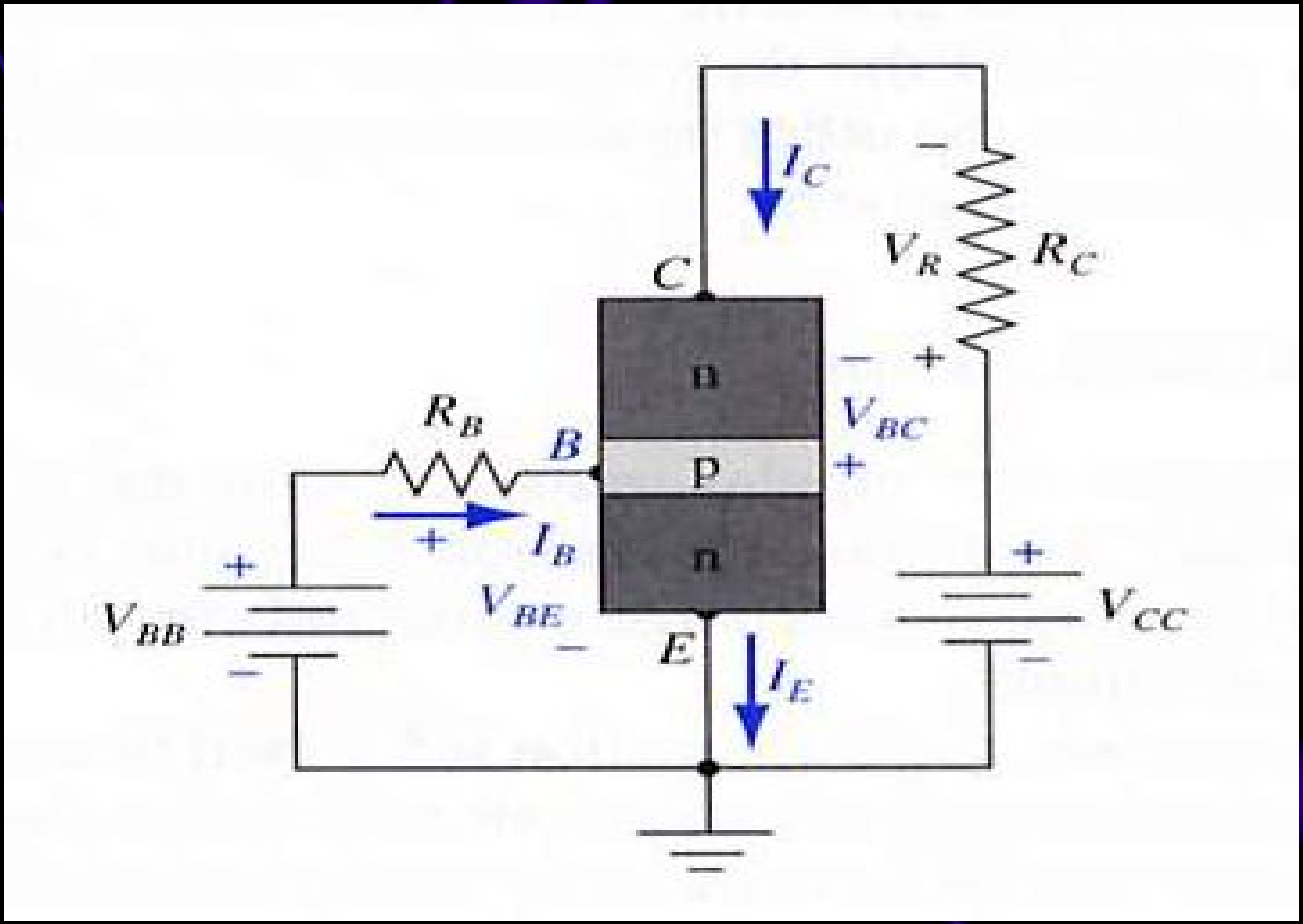
The collector current is being controlled by the B-E voltage, or the current in the one part of the device is being controlled by the voltage in another part - transistor action

Since the B-E junction is forward biased, holes from the base are injected into the emitter. However, these injected holes do not contribute to the collector current and are therefore not part of the transistor action

To design a useful device, we need mathematical expressions for the minority carrier concentrations shown in the figure above.

There are three modes of operation we must consider

- Forward-active (B-E FB, B-C RB)
- Cut-off (B-E RB, B-C RB)
- Saturation (B-E FB, B-C FB)

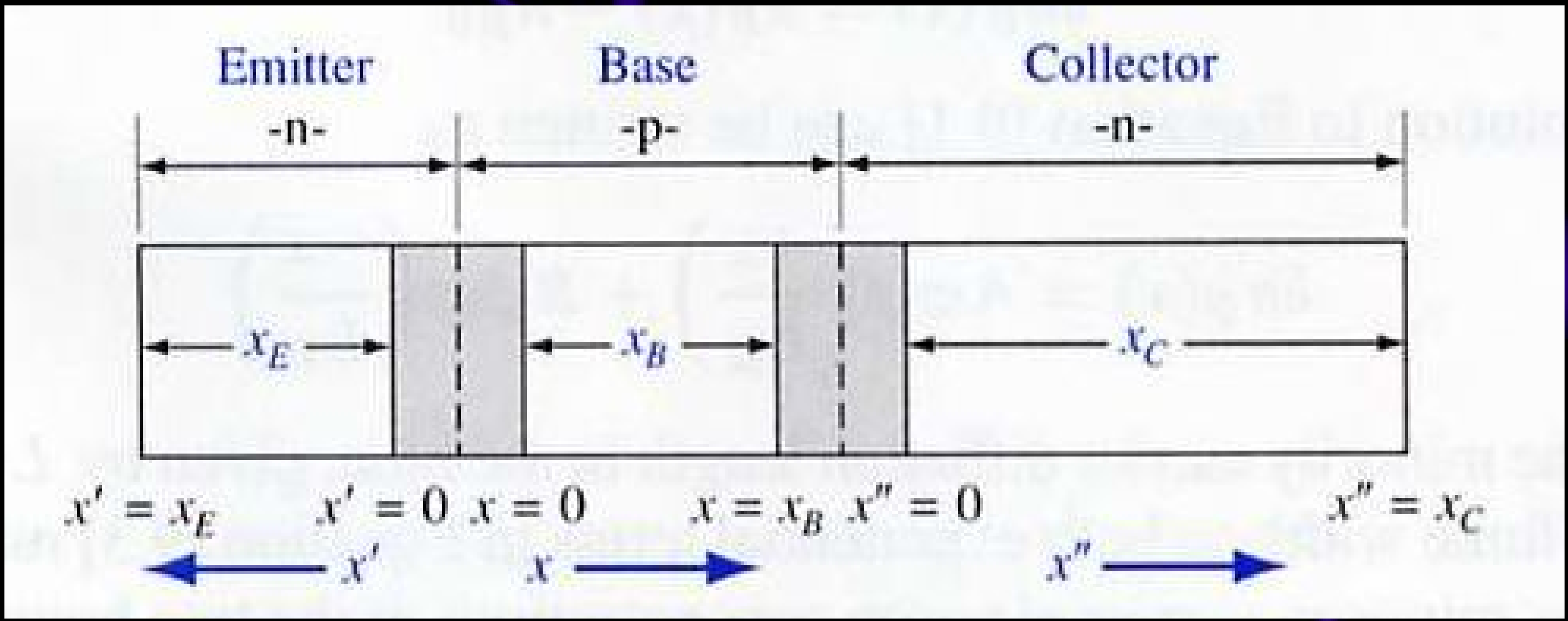


Npn transistor in a simple circuit, known as 'common-emitter'

To calculate the currents in the BJT we must consider minority carrier transport, as we did with the p-n junction diode

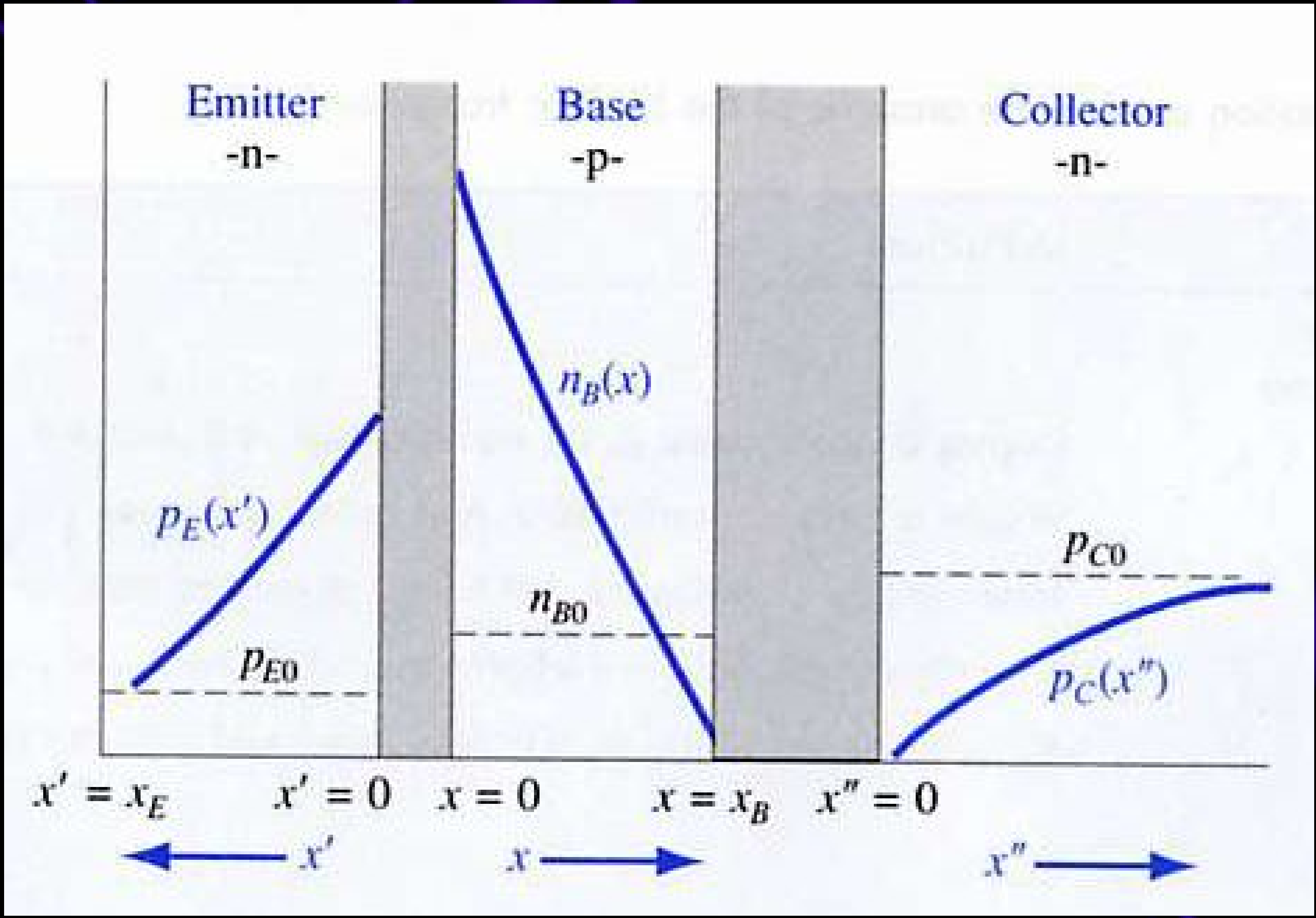
- The table on the next slide shows the notation we are going to use in this analysis

Consider a uniformly doped npn transistor in the forward active mode of operation



Notation	Definition
For Both the NPN and PNP Transistors	
N_E, N_B, N_C	Doping concentrations in the emitter, base, and collector
x_E, x_B, x_C	Widths of neutral emitter, base, and collector regions
D_E, D_B, D_C	*Minority carrier* diffusion coefficients in emitter, base, and collector regions
L_E, L_B, L_C	*Minority carrier* diffusion lengths in emitter, base, and collector regions
τ_{E0}, τ_{B0}, τ_{C0}	*Minority carrier* lifetimes in emitter, base, and collector regions
For the NPN	
p_{E0}, n_{B0}, p_{C0}	Thermal equilibrium *minority carrier* hole, electron, and hole concentrations in the emitter, base, and collector
$p_E(x')$, $n_B(x)$, $p_C(x'')$	Total *minority carrier* hole, electron, and hole concentrations in the emitter base, and collector
$\delta p_E(x')$, $\delta n_B(x)$, $\delta p_C(x'')$	Excess *minority carrier* hole, electron, and hole concentrations in the emitter, base, and collector
For the PNP	
n_{E0}, p_{B0}, n_{C0}	Thermal equilibrium *minority carrier* electron, hole, and electron concentrations in the emitter, base, and collector
$n_E(x')$, $p_B(x)$, $n_C(x'')$	Total *minority carrier* electron, hole, and electron concentrations in the emitter, base, and collector
$\delta n_E(x')$, $\delta p_B(x)$, $\delta n_C(x'')$	Excess *minority carrier* electron, hole, and electron concentrations in the emitter, base, and collector

In the forward active mode, the minority carrier concentrations will appear as



The functions $p_E(x')$, $n_B(x)$ and $p_C(x'')$ denote the steady state minority carrier concentrations in the emitter, base and collector respectively

Assume that the neutral collector length x_C is long Compared to the minority carrier diffusion length L_C in the Collector, but we will take into account a finite emitter Length x_E

Assume that the surface recombination velocity at $x' = x_E$ is Infinite, then the excess minority carrier concentration at $X' = x_E$ is zero, or $p_E\ (x' = x_E) = p_{E0}$

An infinite surface recombination velocity is a good approx. when an ohmic contact is fabricated at $x' = x_E$

We are now ready to analyse the current flows expected in each region of the transistor, and hence be able to model its behaviour during operation

Base region

We need to use the ambipolar transport equation, for a zero electric field in the neutral base region, this equation reduces to

$$D_B \frac{\partial^2(\delta n_B(x))}{\partial x^2} - \frac{\delta n_B(x)}{\tau_{B0}} = 0$$

Where δn_B is the excess minority carrier electron concentration and D_B and τ_{B0} are the minority carrier diffusion coefficent and lifetime in the base region respectively

The excess electron concentration is defined as

$$\delta n_B(x) = n_B(x) - n_{B0}$$

The general solution to the transport equation above can be written as

$$\delta n_B(x) = A \exp\left(\frac{+x}{L_B}\right) + B \exp\left(\frac{-x}{L_B}\right)$$

Where L_B is the minority carrier diffusion length in the base, given by

$$L_B = \sqrt{D_B \tau_{B0}}.$$

The excess minority carrier electron concentrations at the two boundaries become

$$\delta n_B(x = 0) \equiv \delta n_B(0) = A + B$$

and

$$\delta n_B(x = x_B) \equiv \delta n_B(x_B) = A\exp\left(\frac{+x_B}{L_B}\right) + B\exp\left(\frac{-x_B}{L_B}\right)$$

The B-E junction is forward biased, so the boundary condition at x=0 is

$$\delta n_B(0) = n_B(x=0) - n_{B0} = n_{B0}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]$$

The B-C junction is reverse biased, so the second boundary condition at x = x_B is

$$\delta n_B(x_B) = n_B(x = x_B) - n_{B0} = 0 - n_{B0} = -n_{B0}$$

Combining these equations enables the coefficeints A and B to be determined, resulting in

$$A = \frac{-n_{B0} - n_{B0}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]\exp\left(\frac{-x_B}{L_B}\right)}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

$$B = \frac{n_{B0}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]\exp\left(\frac{x_B}{L_B}\right) + n_{B0}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

Substitution into the very first (transport) equation then gives the excess minority carrier electron concentration

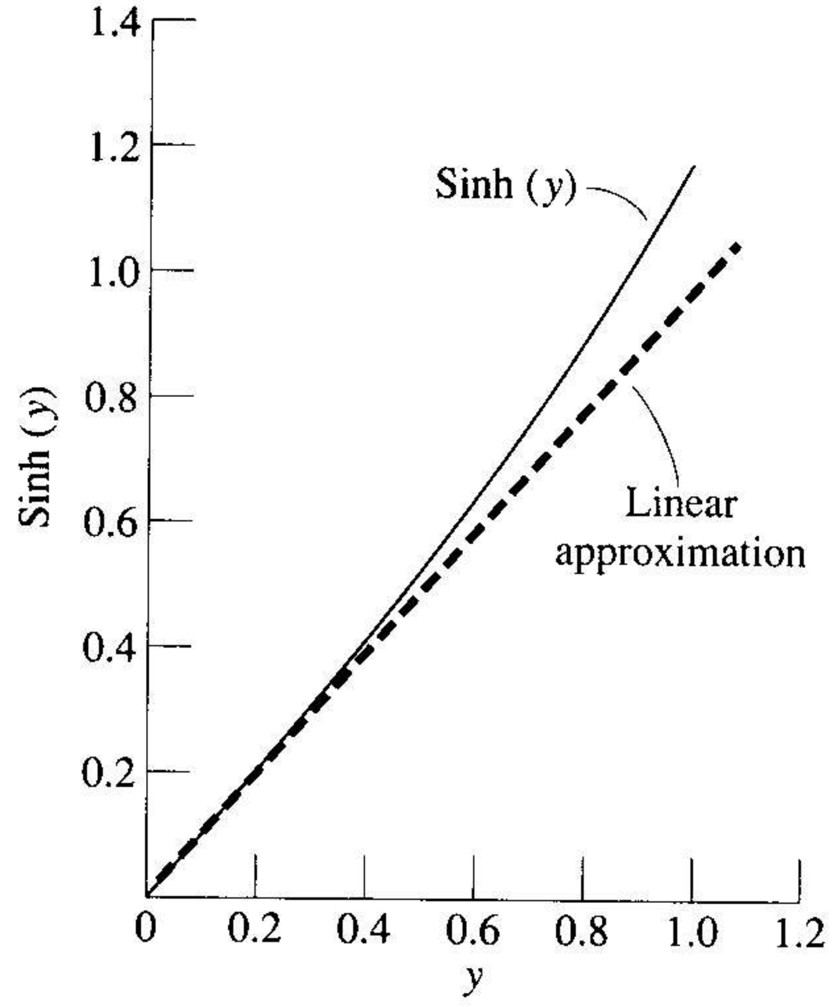
$$\delta n_B(x) = \frac{n_{B0}\left\{\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]\sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)\right\}}{\sinh\left(\frac{x_B}{L_B}\right)}$$

Looks horrible with the sinh functions!

We want the base width x_B to be small compared to the Minority carrier diffusion length L_B

Since $x_B < L_B$, the argument in the sinh functions is always less than unity, mostly very much less

To make our equation easier to handle, we can look at ways to simplify out the sinh functions



Hyperbolic sine function and its linear approximation

If y < 0.4, the sinh(y) function is within 3% of the linear approx

This implies that we can Use the approx

Sinh(x) = x

for x <<1

we then get,

$$\delta n_B(x) \cong \frac{n_{B0}}{x_B}\left\{\left[\exp\left(\frac{eV_{BE}}{kT}\right)-1\right](x_B - x) - x\right\}$$

Emitter region

Consider the minority hole concentration in the emitter
The steady state excess hole concentration is determined from

$$D_E\frac{\partial^2(\delta p_E(x'))}{\partial x^2} - \frac{\delta p_E(x')}{\tau_{E0}} = 0$$

Where D_E and τ_{E0} are the minority carrier diffusion co-eff and lifetime respectively, in the emitter. The excess hole concentration is given by,

$$\delta p_E(x') = p_E(x') - p_{E0}$$

The general solution to the steady state equation above is

$$\delta p_E(x') = C \exp\left(\frac{+x'}{L_E}\right) + D \exp\left(\frac{-x'}{L_E}\right)$$

where

$$L_E = \sqrt{D_E \tau_{E0}}.$$

The excess minority carrier hole concentrations at the two boundaries are,

$$\delta p_E(x' \geq 0) \equiv \delta p_E(0) = C + D$$

and

$$\delta p_E(x' = x_E) \equiv \delta p_E(x_E) = C \exp\left(\frac{x_E}{L_E}\right) + D \exp\left(\frac{-x_E}{L_E}\right)$$

Again, the B-E junction is forward biased so

$$\delta p_E(0) = p_E(x' = 0) - p_{E0} = p_{E0}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]$$

An infinite surface recombination velocity at $x' = x_E$ implies

$$\delta p_E(x_E) = 0$$

Solving for C and D, the excess minority carrier concentration Becomes

$$\delta p_E(x') = \frac{p_{E0}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]\sinh\left(\frac{x_E - x'}{L_E}\right)}{\sinh\left(\frac{x_E}{L_E}\right)}$$

We can again use the linear approx for the sinh terms if x_E is small and hence

$$\delta p_E(x') = \frac{p_{E0}}{x_E}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right](x_E - x')$$

If x_E is comparable to L_E, then $\delta p_E(x')$ shows an exponential dependence on x_E

Collector region

The excess minority carrier hole concentration in the collector can be determined from,

$$D_C \frac{\partial^2(\delta p_C(x''))}{\partial x''^2} - \frac{\delta p_C(x'')}{\tau_{C0}} = 0$$

Where D_C and τ_{C0} are the minority carrier diffusion coefficent and lifetime respectively. The excess minority hole concentration in the collector will be,

$$\delta p_C(x'') = p_C(x'') - p_{C0}$$

and a general solution to the transport equation above can be determined as,

$$\delta p_C(x'') = G \exp\left(\frac{x''}{L_C}\right) + H \exp\left(\frac{-x''}{L_C}\right)$$

where

$$L_C = \sqrt{D_C \tau_{C0}}.$$

If we assume that the collector is long, then the coefficient G must be zero since the excess concentration must remain finite. The second boundary condition gives,

$$\delta p_C(x''=0) \equiv \delta p_C(0) = p_C(x''=0) - p_{C0} = 0 - p_{C0} = -p_{C0}$$

The excess minority carrier hole concentration in the collector is then given by,

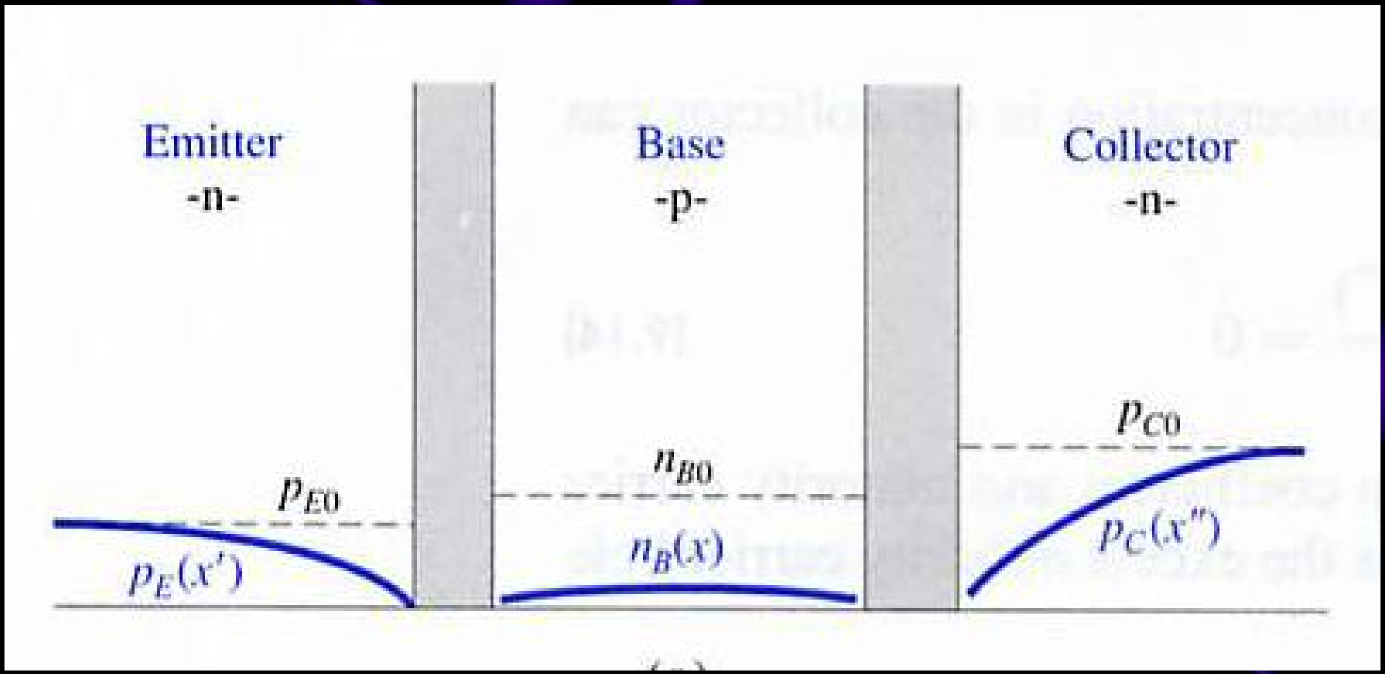
$$\boxed{\delta p_c(x'') = -p_{C0}\exp\left(\frac{-x''}{L_C}\right)}$$

This result is the same as you found in the first half of the course for a reverse-biased pn junction

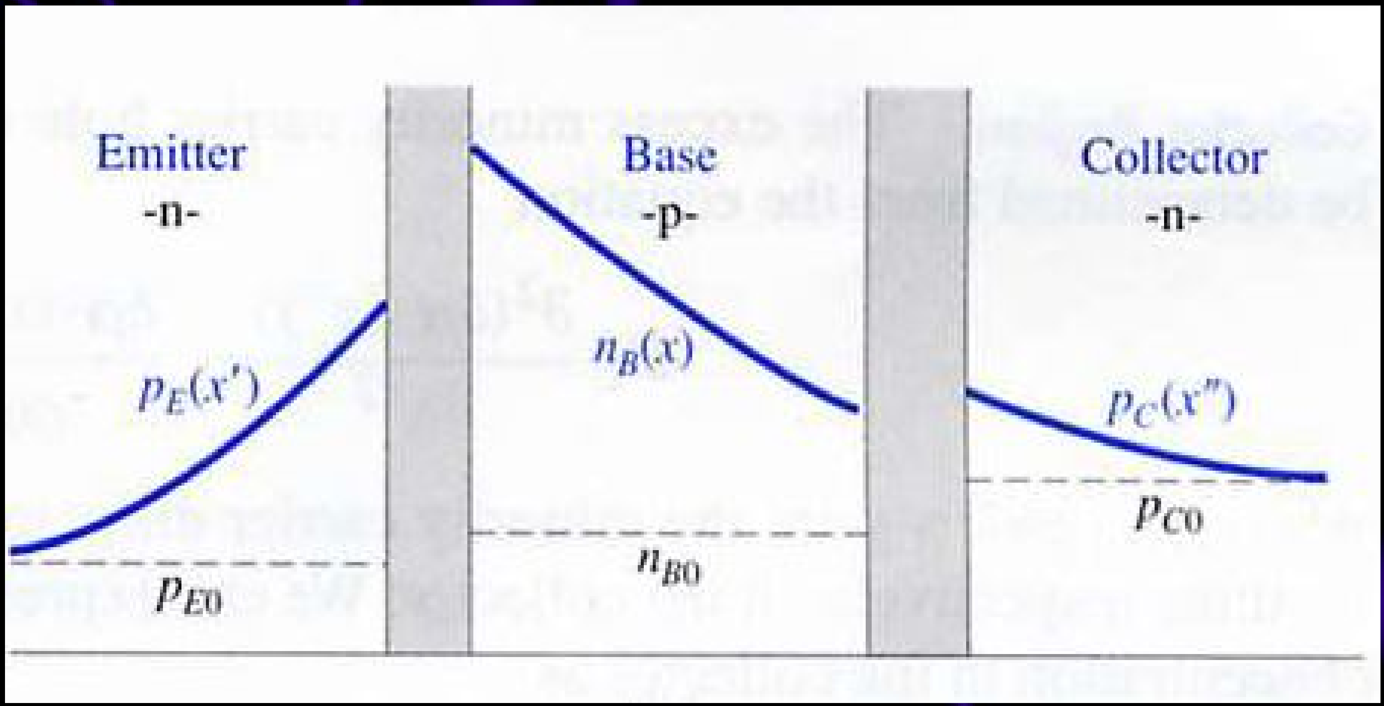
Other modes of operation

You can now understand for yourself the minority carrier Concentrations that we would expect for the three other modes

- Cut-off



- Saturation



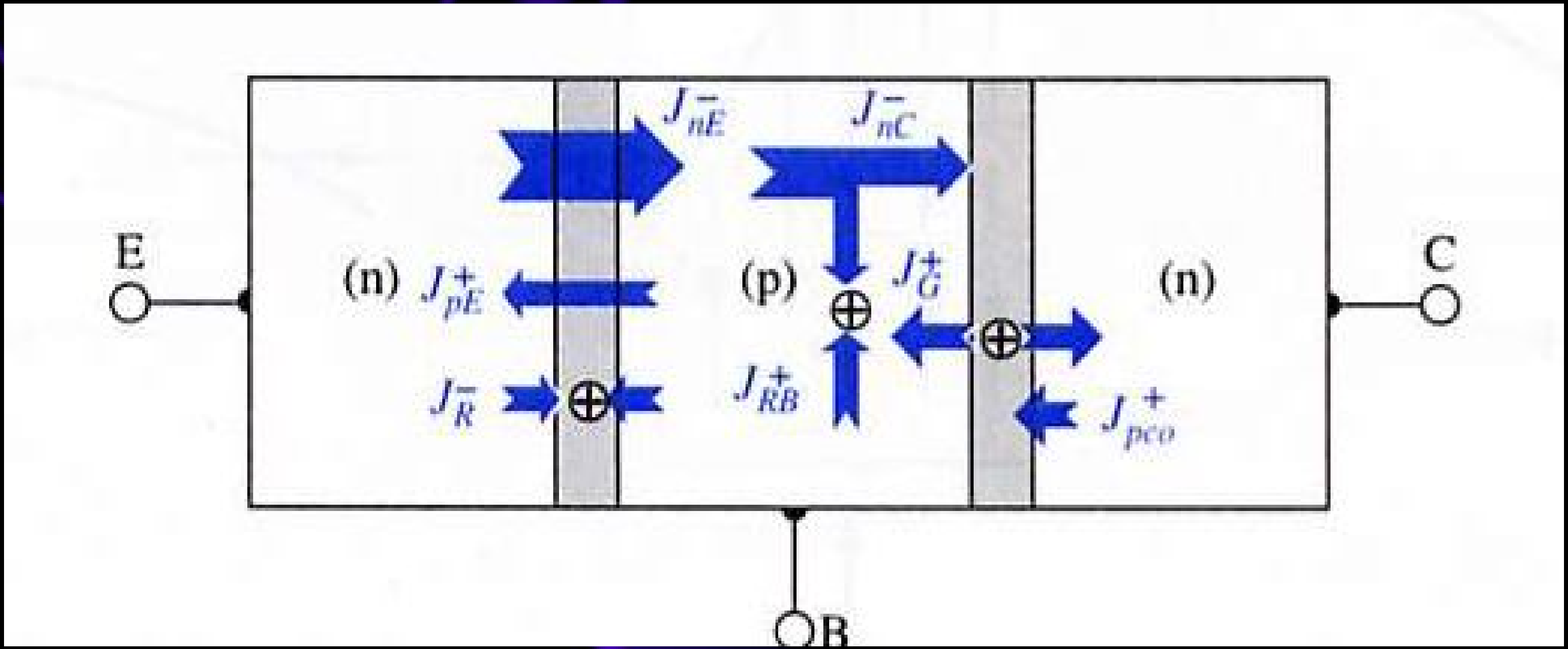
Low frequency common base current gain

The basic operation of the BJT involves the control of the collector current by the B-E voltage

The collector current is a function of the number of majority carriers reaching the collector after being injected from the emitter across the B-E junction

The *common-base current gain* is defined as the ratio of Collector current to emitter current

- We need to consider the current terms involved in this value
- Begin by considering the various flux components in the npn transistor



Question

Name, and explain the origin of, each of the terms in the diagram above

J_{nE}: Due to the diffusion of minority carrier electrons in the base at $x = 0$.

J_{nC}: Due to the diffusion of minority carrier electrons in the base at $x = x_B$.

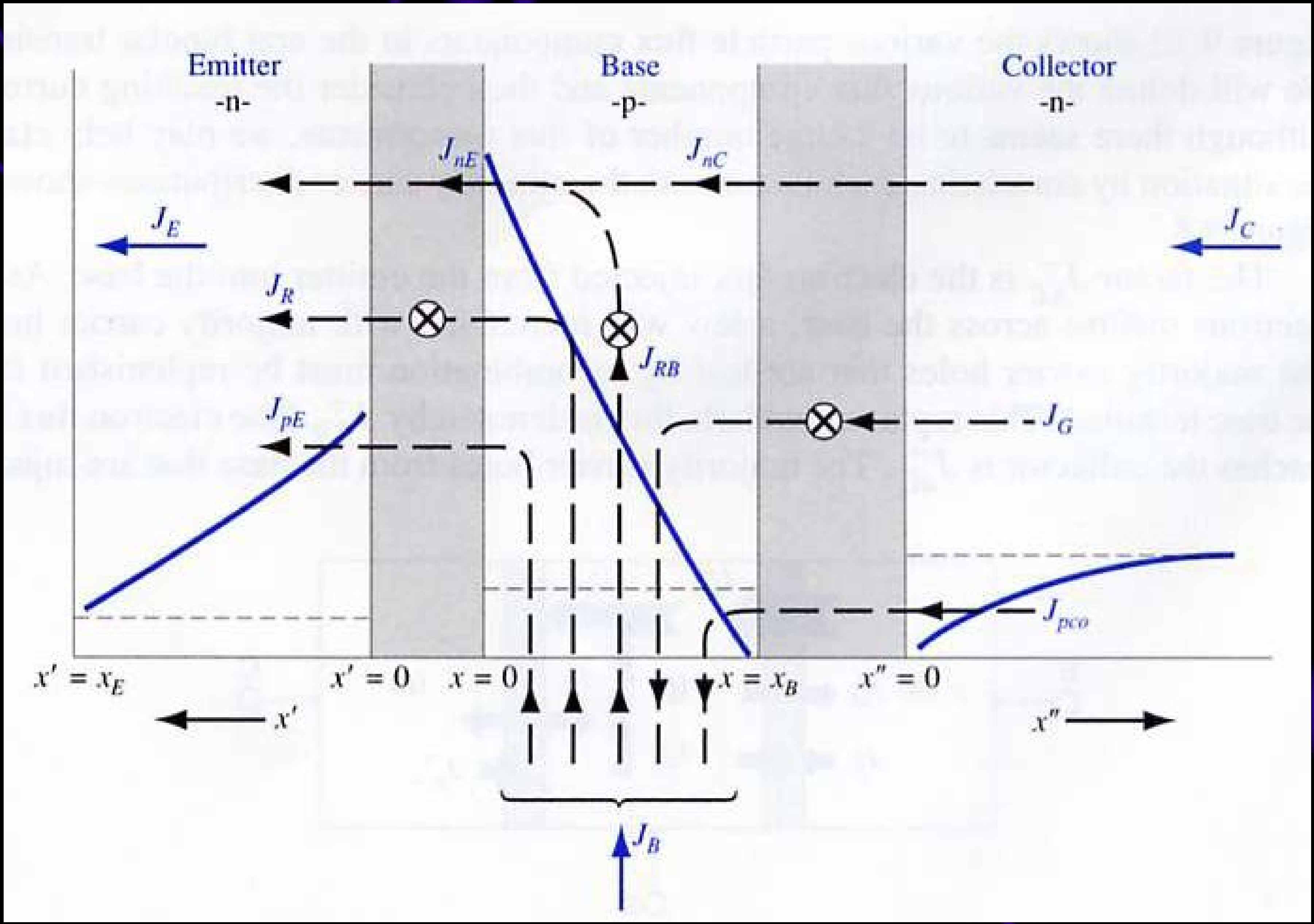
J_{RB}: The difference between J_{nE} and J_{nC}, which is due to the recombination of excess minority carrier electrons with majority carrier holes in the base. The J_{RB} current is the flow of holes into the base to replace the holes lost by recombination.

J_{pE}: Due to the diffusion of minority carrier holes in the emitter at $x' = 0$.

J_R: Due to the recombination of carriers in the forward-biased B-E junction.

J_{pc0}: Due to the diffusion of minority carrier holes in the collector at $x'' = 0$.

J_G: Due to the generation of carriers in the reverse-biased B-C junction.



Emitter
-n-
Base
-p-
Collector
-n-
J_{nE}
J_{nC}
J_E
J_C
J_R
J_{RB}
J_{pE}
J_G
J_{pco}
$x' = x_E$
$x' = 0$
$x = 0$
$x = x_B$
$x'' = 0$
x'
x''
J_B

The currents J_{RB}, J_{pE} and J_R are B-E junction currents and do not contribute to the collector current. The currents J_{pc0} and J_G are B-C junction currents only. These current components do not contribute to the transistor action or the current gain.

The dc common base current gain is defined as

$$\alpha_0 = \frac{I_C}{I_E}$$

If we assume that the active cross-sectional area is the same for the collector and emitter, then

$$\alpha_0 = \frac{J_C}{J_E} = \frac{J_{nC} + J_G + J_{pc0}}{J_{nE} + J_R + J_{pE}}$$

We are primarily interested in finding how the collector current will change with a change in the emitter current. The small signal, or sinusoidal, common base current gain is defined as

$$\alpha = \frac{\partial J_C}{\partial J_E} = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}}$$

This can be re-written as

$$\alpha = \left(\frac{J_{nE}}{J_{nE} + J_{pE}}\right)\left(\frac{J_{nC}}{J_{nE}}\right)\left(\frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}}\right)$$

$$\alpha = \gamma \alpha_T \delta \quad \text{Equation 1}$$

Where

$$\gamma = \left(\frac{J_{nE}}{J_{nE} + J_{pE}}\right) \equiv \text{Emitter injection efficiency factor}$$

$$\alpha_T = \left(\frac{J_{nC}}{J_{nE}}\right) \equiv \text{Base transport factor}$$

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \equiv \text{Recombination factor}$$

We would like to have the change in the collector current to be the same as the change in the emitter current, ie α = 1. To get close to this each term equation 1 above must be as close to 1 as possible.

The *emitter injection efficiency factor* takes into account the minority hole diffusion current in the emitter. This current is part of the emitter current, but does not contribute to the transistor action in that J_{pE} is not part of the collector current.

The *base transport factor* takes into account any recombination of excess minority carrier electrons in the base. Ideally we want no recombination.

The *recombination factor* takes into account the recombination in the forward biased B-E junction. This current, J_R, contributes to the emitter current but does not contribute to the collector current.

We now need to determine each of the gain factors in terms of the electrical and geometrical parameters of the transistor - this will enable us to design the effective BJTs

Emitter injection efficiency factor

$$\gamma = \left(\frac{J_{nE}}{J_{nE} + J_{pE}}\right) = \frac{1}{\left(1 + \frac{J_{pE}}{J_{nE}}\right)}$$

We can write the current densities as

$$J_{pE} = -eD_E \frac{d(\delta p_E(x'))}{dx'}\bigg|_{x'=0}$$

$$J_{nE} = (-)eD_B \frac{d(\delta n_B(x))}{dx}\bigg|_{x=0}$$

Taking derivatives then gives us

$$J_{pE} = \frac{eD_E p_{E0}}{L_E}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right] \cdot \frac{1}{\tanh(x_E/L_E)}$$

$$J_{nE} = \frac{eD_B n_{B0}}{L_B}\left\{\frac{1}{\sinh(x_B/L_B)} + \frac{[\exp(eV_{BE}/kT) - 1]}{\tanh(x_B/L_B)}\right\}$$

If we assume that the B-E junction is sufficiently far in
In the forward bias regime so that $V_{BE} >> kT/e$, then

$$\exp\left(\frac{eV_{BE}}{kT}\right) \gg 1$$

and also

$$\frac{\exp(eV_{BE}/kT)}{\tanh(x_B/L_B)} \gg \frac{1}{\sinh(x_B/L_B)}$$

The emitter injection efficiency then becomes

$$\boxed{\gamma = \frac{1}{1 + \frac{p_{E0} D_E L_B}{n_{B0} D_B L_E} \cdot \frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}}$$

If we assume that all of the parameters in this equation are fixed, except for p_{E0} and n_{B0}, then in order that γ is close to unity, we must have $p_{E0} << n_{B0}$. We can write,

$$p_{E0} = \frac{n_i^2}{N_E} \qquad \text{and} \qquad n_{B0} = \frac{n_i^2}{N_B}$$

where N_E and N_B are the impurity doping concentrations in the emitter and base. This then implies that $N_E >> N_B$.

If both $x_B << L_B$ and $x_E << L_E$ then the emitter injection efficiency can be written as

$$\boxed{\gamma \cong \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}}}$$

Base Transport factor

From the definitions of the current directions we used above we can write,

$$J_{nC} = (-)eD_B \left.\frac{d(\delta n_B(x))}{dx}\right|_{x=x_B}$$

$$J_{nE} = (-)eD_B \left.\frac{d(\delta n_B(x))}{dx}\right|_{x=0}$$

If we then consider the expression we had for $\delta n_B(x)$ before, we find that

$$J_{nC} = \frac{eD_B n_{B0}}{L_B}\left\{\frac{[\exp(eV_{BE}/kT) - 1]}{\sinh(x_B/L_B)} + \frac{1}{\tanh(x_B/L_B)}\right\}$$

Again assume that the B-E junction is biased sufficiently far in the forward region so that $V_{BE} >> kT/e$, then $\exp(eV_{BE}/kT) >> 1$, substitution then gives,

$$\alpha_T = \frac{J_{nC}}{J_{nE}} \cong \frac{\exp(eV_{BE}/kT) + \cosh(x_B/L_B)}{1 + \exp(eV_{BE}/kT)\cosh(x_B/L_B)}$$

For α_T to be close to unity, the neutral base width x_B must be much smaller than the minority carrier diffusion length in the base L_B.

If $x_B << L_B$, then $\cosh(x_B/L_B)$ will be just slightly greater than unity, and if $\exp(eV_{BE}/kT) >> 1$, the base transport factor will approximate to,

$$\boxed{\alpha_T \cong \frac{1}{\cosh(x_B/L_B)}}$$

For $x_B << L_B$, we can expand the cosh function in a Taylor series, so that

$$\boxed{\alpha_T = \frac{1}{\cosh(x_B/L_B)} \approx \frac{1}{1 + \frac{1}{2}(x_B/L_B)^2} \approx 1 - \frac{1}{2}(x_B/L_B)^2}$$

So, the base transport factor will be close to one if $x_B << L_B$

Recombination factor

If we assume $J_{pE} << J_{nE}$, then we can write

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \cong \frac{J_{nE}}{J_{nE} + J_R} = \frac{1}{1 + J_R / J_{nE}}$$

The recombination current density due to the recombination in a forward biased p-n junction is as derived earlier in the course, and can be written as,

$$J_R = \frac{e x_{BE} n_i}{2\tau_0} \exp\left(\frac{eV_{BE}}{2kT}\right) = J_{r0} \exp\left(\frac{eV_{BE}}{2kT}\right)$$

where x_{BE} is the B-E space charge width

The current J_{nE} can be approximated to

$$J_{nE} = J_{s0} \exp\left(\frac{eV_{BE}}{kT}\right)$$

where

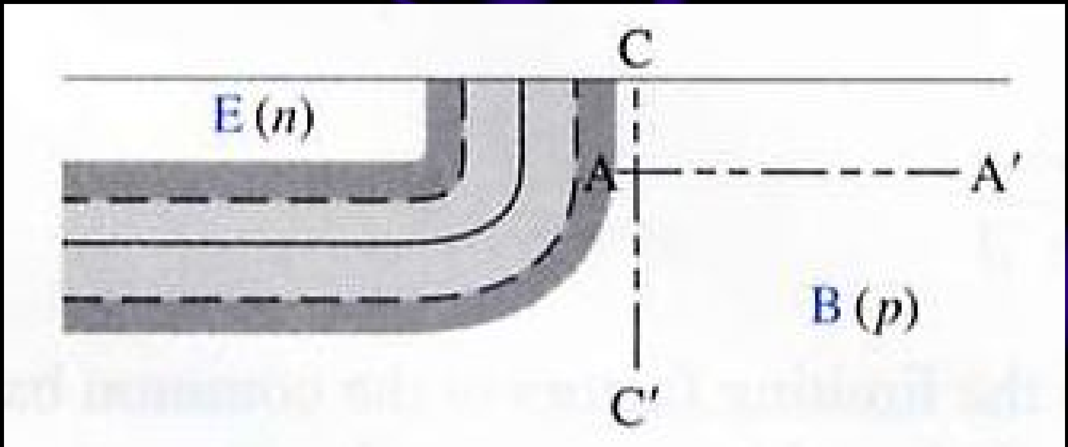
$$J_{s0} = \frac{eD_B n_{B0}}{L_B \tanh(x_B/L_B)}$$

The recombination factor can then be written as

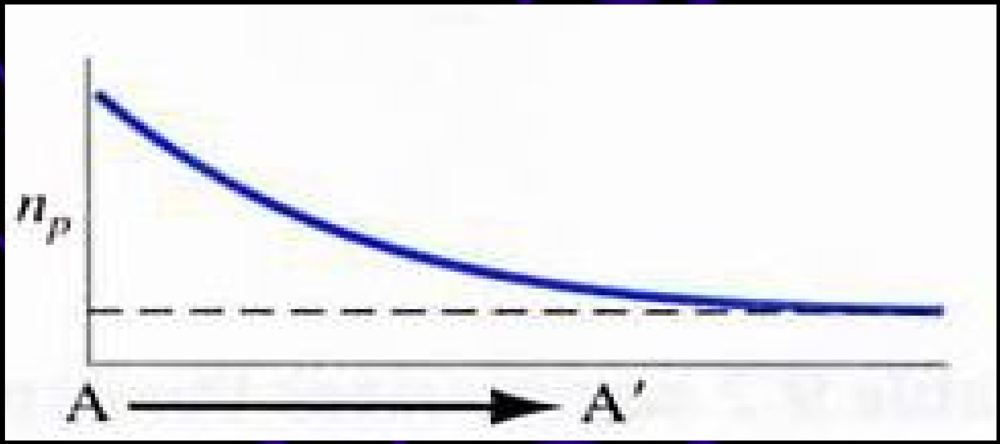
$$\delta = \frac{1}{1 + \frac{J_{r0}}{J_{s0}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

The recombination factor is a function of the B-E voltage. As V_{BE} increases, the recombination current becomes less dominant and the recombination factor approaches unity

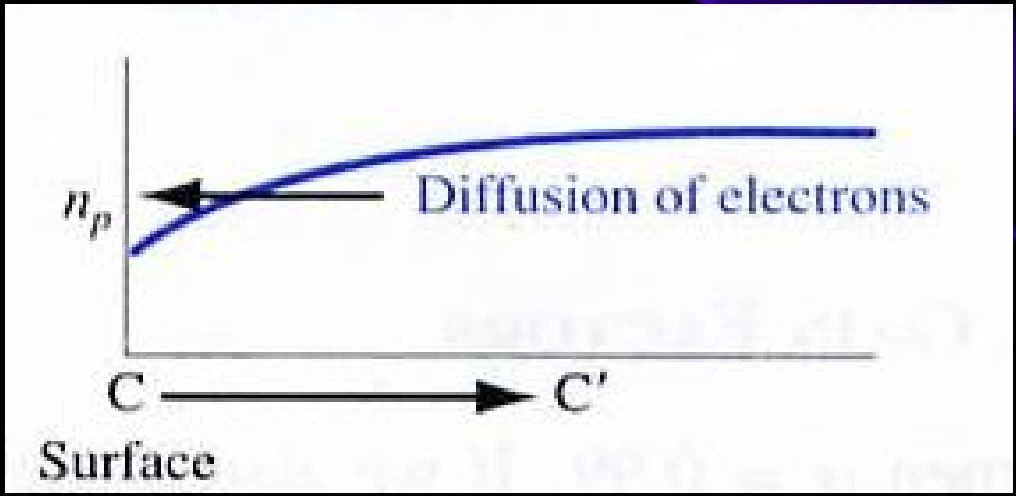
The recombination factor must also include surface effects; the figure below shows the B-E junction of an npn transistor near the semiconductor surface



Assume the B-E junction is forward biased - the excess minority carrier concentration in the base along the cross section A-A’ will be,

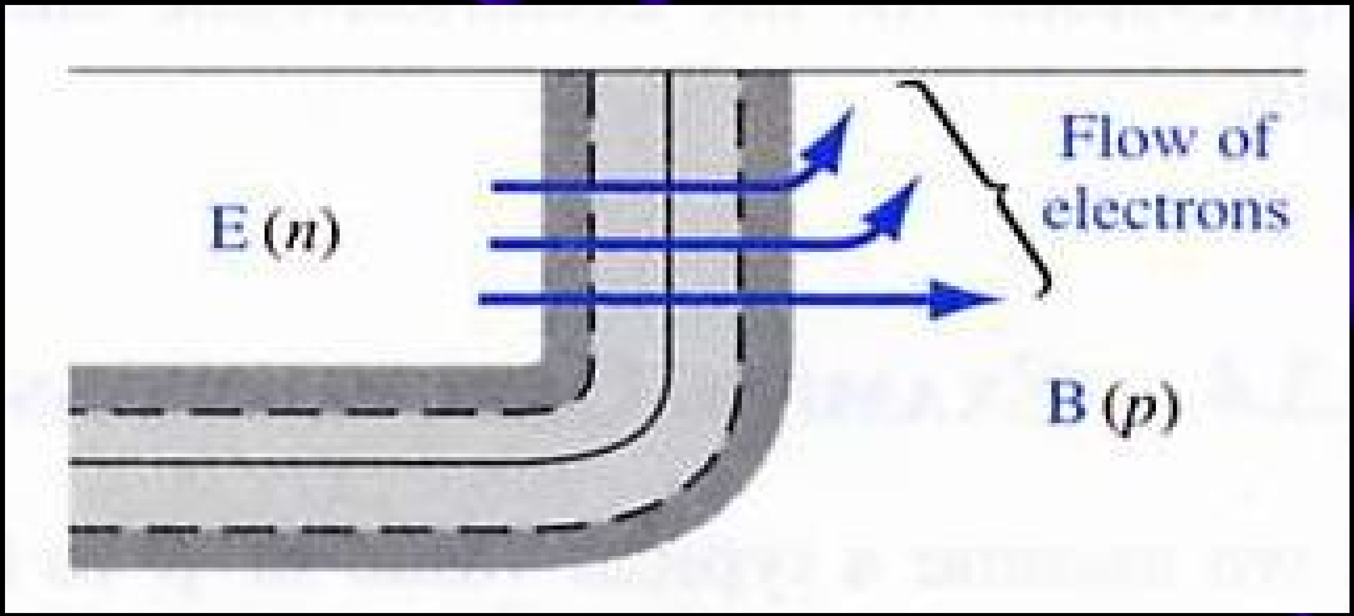


This curve represents the normal forward-biased junction minority carrier concentration. However, the cross section at C-C' from the surface will be,



The excess concentration at the surface is smaller than the excess concentration in the bulk - this causes an electron Diffusion current to occur from the bulk to the surface, where the electrons recombine with the majority carrier holes.

This additional recombination current must be included in the recombination factor δ - difficult to calculate due to the 2-D nature



Although the have been considering an npn transistor, the same analysis for all of these transport factors applies to pnp devices

We have also been considering the common base current gain,

$$\alpha_0 = I_C / I_E .$$

but we can also consider the common emitter current gain which is defined as

$$\beta_0 = I_C / I_B$$

The relation between the common base and common emitter current gains comes from,

$$\frac{1}{\alpha_0} = \frac{1}{\beta_0} + 1$$

Since this relationship holds for both dc and small-signal conditions, we can drop the subscript and write,

$$\beta = \frac{\alpha}{1-\alpha}$$

and then the common base current gain in terms of the common emitter current gain is found to be,

$$\alpha = \frac{\beta}{1+\beta}$$

Emitter injection efficiency

$$\gamma \cong \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}} \qquad (x_B \ll L_B), (x_E \ll L_E)$$

Base transport factor

$$\alpha_T \cong \frac{1}{1 + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2} \qquad (x_B \ll L_B)$$

Recombination factor

$$\delta = \frac{1}{1 + \frac{J_{ro}}{J_{so}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

Common-base current gain

$$\alpha = \gamma \alpha_T \delta \cong \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E} + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2 + \frac{J_{ro}}{J_{so}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

Common-emitter current gain

$$\beta = \frac{\alpha}{1 - \alpha} \cong \frac{1}{\frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E} + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2 + \frac{J_{ro}}{J_{so}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

If we assume a typical values of β to be 100, then α will equal 0.99. If we also assume,

$$\gamma = \alpha_T = \delta$$

then each factor would have to be equal to 0.9967 to get a β value of 100

This simple calculation indicates just how close to unity each factor must be in order to achieve a reasonable current gain

Questions

Give a qualitative explanation of current gain in a BJT

Describe the key device design factors involved in achieving high current gain in a BJT

Qualitative arguments

We can see that a current of electrons flows out of the emitter and if only a few of these recombine in the base then we will have a collector current of almost the same magnitude (forward active mode)

- There will also be a small current of holes flowing into the base from the external contact, to account for charge neutrality as some recombination does occur

The key to using the device as a amplifier is to use this base current to control the collector current

Consider the effect of applying a small current of holes to the base

- These excess holes will introduce a small positive charge into the base

- Electrons must therefore flow from the emitter to the base to counter this current (can not be from the collector as this junction is reverse biased)

- Since most of the electrons from the emitter flow through the base without recombination, the number electrons flowing from the emitter must greatly exceed the number of holes flowing into the base

- For example, if we assume only 1% of electrons recombine with a hole, then 99% continue into the collector; if we vary the base current, the current of electrons flowing from the emitter to the collector varies accordingly, in direct proportion to the current flowing in at the base

- The difference is that the collector current is much larger than the base current, here 99x larger

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Hence, if we use this device in a situation where the base current serves as the input and the collector current as the output, then the transistor acts as an amplifier

But - how is it that we can convert a weak signal into a Strong one? Normally, there is no ‘free lunch’

- the large current is already present - it is the current applied to the emitter of the transistor
- the function of the transistor is simply to ‘imprint’ the pattern of the weak signal onto this large current to produce a much stronger signal

This leads us to the limitation on this type of device

- If the base current is too large then the voltage across the emitter will not be sufficient to supply the required number of electrons SATURATION

Example 1

Design the ratio of emitter doping to base doping in a BJT to achieve an emitter injection efficiency factor equal to γ= 0.9967 (npn device)

For simplicity assume

$D_E = D_B$, $L_E = L_B$ and $x_E = x_B$

From the equations above we have,

$$\gamma = \frac{1}{1 + \dfrac{p_{E0}}{n_{B0}}} = \frac{1}{1 + \dfrac{n_i^2/N_E}{n_i^2/N_B}}$$

So,

$$\gamma = \frac{1}{1 + \dfrac{N_B}{N_E}} = 0.9967$$

then,

$$\frac{N_B}{N_E} = 0.00331 \quad \text{or} \quad \frac{N_E}{N_B} = 302$$

Hence, the emitter doping concentration must be much larger than the base doping concentration to achieve a high emitter injection efficiency

Example 2

Design the base width required to achieve a base transport factor equal to α_T = 0.9967 (pnp device)

Assume that D_B = 10 cm^2/sec and τ_{B0} equals 10^{-7} sec

The base transport factor is given by,

$$\alpha_T = \frac{1}{\cosh(x_B/L_B)} = 0.9967$$

$$x_B/L_B = 0.0814$$

$$L_B = \sqrt{D_B \tau_{B0}} = \sqrt{(10)(10^{-7})} = 10^{-3} \text{ cm}$$

$$x_B = 0.814 \times 10^{-4} \text{ cm} = 0.814 \ \mu\text{m}$$

If the base width is less than approx. 0.8μm, then the required base transport factor will be achieved.

In most cases, the base transport factor will not be the limiting factor in the BJT current gain

Example 3

Calculate the forward-biased B-E voltage required to achieve a recombination factor equal to 0.9967.

Consider an npn transistor at 300K. Assume that $J_{S0} = 10^{-11}$ A/cm^2

$$\delta = \frac{1}{1 + \frac{J_{r0}}{J_{s0}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

We then have

$$0.9967 = \frac{1}{1 + \frac{10^{-8}}{10^{-11}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

Re-arranging this gives

$$\exp\left(\frac{+eV_{BE}}{2kT}\right) = \frac{0.9967 \times 10^{3}}{1 - 0.9967} = 3.02 \times 10^{5}$$

$$V_{BE} = 2(0.0259)\ln(3.02 \times 10^5) = 0.654 \text{ volt}$$

This example demonstrates that the recombination factor may be an important limiting factor in the BJT gain. Here, if V_{BE} is smaller than 0.654V, then the recombination factor γ will fall below the desired 0.9967 value.

Example 4

Calculate the common-emitter current gain of a silicon npn BJT, taking T = 300K.

Assume the following parameters,

$$D_E = 10 \text{ cm}^2/\text{sec} \qquad x_B = 0.70\ \mu\text{m}$$
$$D_B = 25 \text{ cm}^2/\text{sec} \qquad x_E = 0.50\ \mu\text{m}$$
$$\tau_{E0} = 1 \times 10^{-7} \text{ sec} \qquad N_E = 1 \times 10^{18} \text{ cm}^{-3}$$
$$\tau_{B0} = 5 \times 10^{-7} \text{ sec} \qquad N_B = 1 \times 10^{16} \text{ cm}^{-3}$$
$$J_{r0} = 5 \times 10^{-8} \text{ A/cm}^2 \qquad V_{BE} = 0.65 \text{ volt}$$

We can calculate,

$$p_{E0} = \frac{(1.5 \times 10^{10})^2}{1 \times 10^{18}} = 2.25 \times 10^2 \text{ cm}^{-3}$$
$$n_{B0} = \frac{(1.5 \times 10^{10})^2}{1 \times 10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$L_E = \sqrt{D_E \tau_{E0}} = 10^{-3}\ \text{cm}$$
$$L_B = \sqrt{D_B \tau_{B0}} = 3.54 \times 10^{-3}\ \text{cm}$$

The emitter injection efficiency factor will be,

$$\gamma = \frac{1}{1 + \dfrac{(2.25 \times 10^2)(10)(3.54 \times 10^{-3})}{(2.25 \times 10^4)(25)(10^{-3})} \cdot \dfrac{\tanh(0.0198)}{\tanh(0.050)}} = 0.9944$$

We now need to calculate the base transport factor, again using the equation we derived earlier, and then the recombination factor

$$\alpha_T = \frac{1}{\cosh\left(\dfrac{0.70 \times 10^{-4}}{3.54 \times 10^{-3}}\right)} = 0.9998$$

$$\delta = \frac{1}{1 + \dfrac{5 \times 10^{-8}}{J_{s0}} \exp\left(\dfrac{-0.65}{2(0.0259)}\right)}$$

Where J_{S0} can be calculated from,

$$J_{s0} = \frac{eD_B n_{B0}}{L_B \tanh\left(\dfrac{x_B}{L_B}\right)} = \frac{(1.6 \times 10^{-19})(25)(2.25 \times 10^{4})}{3.54 \times 10^{-3} \tanh(1.977 \times 10^{-2})} = 1.29 \times 10^{-9}\ \text{A/cm}^2$$

We can now calculate that δ = 0.99986. The common-base current gain is then,

$$\alpha = \gamma \alpha_T \delta = (0.9944)(0.9998)(0.99986) = 0.99406$$

which gives a common-emitter current gain of

$$\beta = \frac{\alpha}{1-a} = \frac{0.99406}{1-0.99406} = 167$$

In this example we can see that the emitter injection efficiency is the limiting factor in the current gain level we can achieve

Real BJTs - the influence of non-ideality

We have been considering a BJT which has

- Uniform doped regions
- Low injection
- Constant emitter and base widths
- An ideal constant energy band-gap
- Uniform current densities
- The absence of junction breakdown

If any of these conditions are not met, then the BJT properties will deviate from those we have been calculating

It is therefore important to consider the likely effect of deviation from the ideal in each case

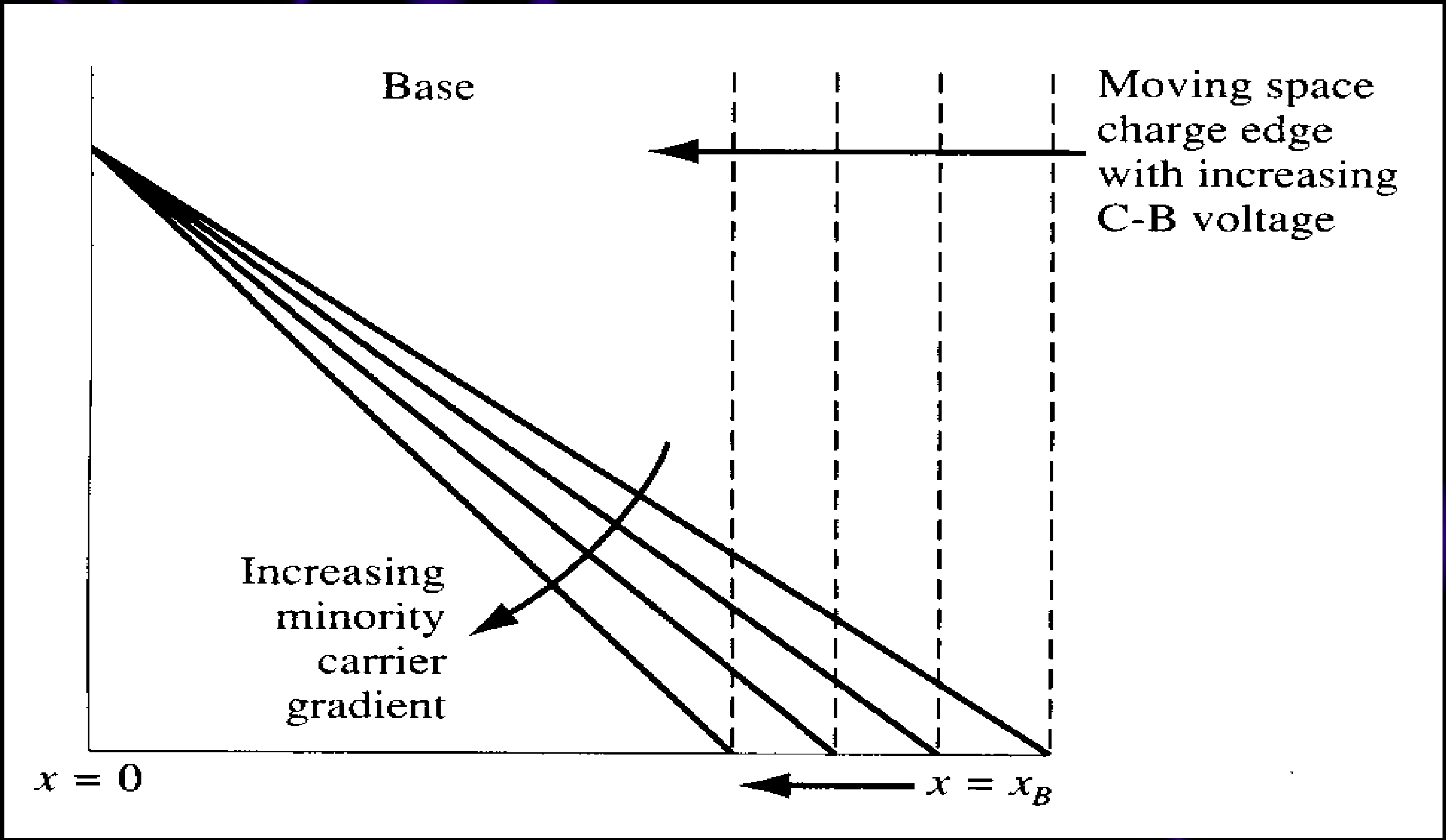
Base width modulation - *the Early effect*

We have been assuming that the neutral base width x_B is constant. However, in practice it will be a function of the B-C junction voltage, since the width of the space charge region extending into the base varies with V_{BC}

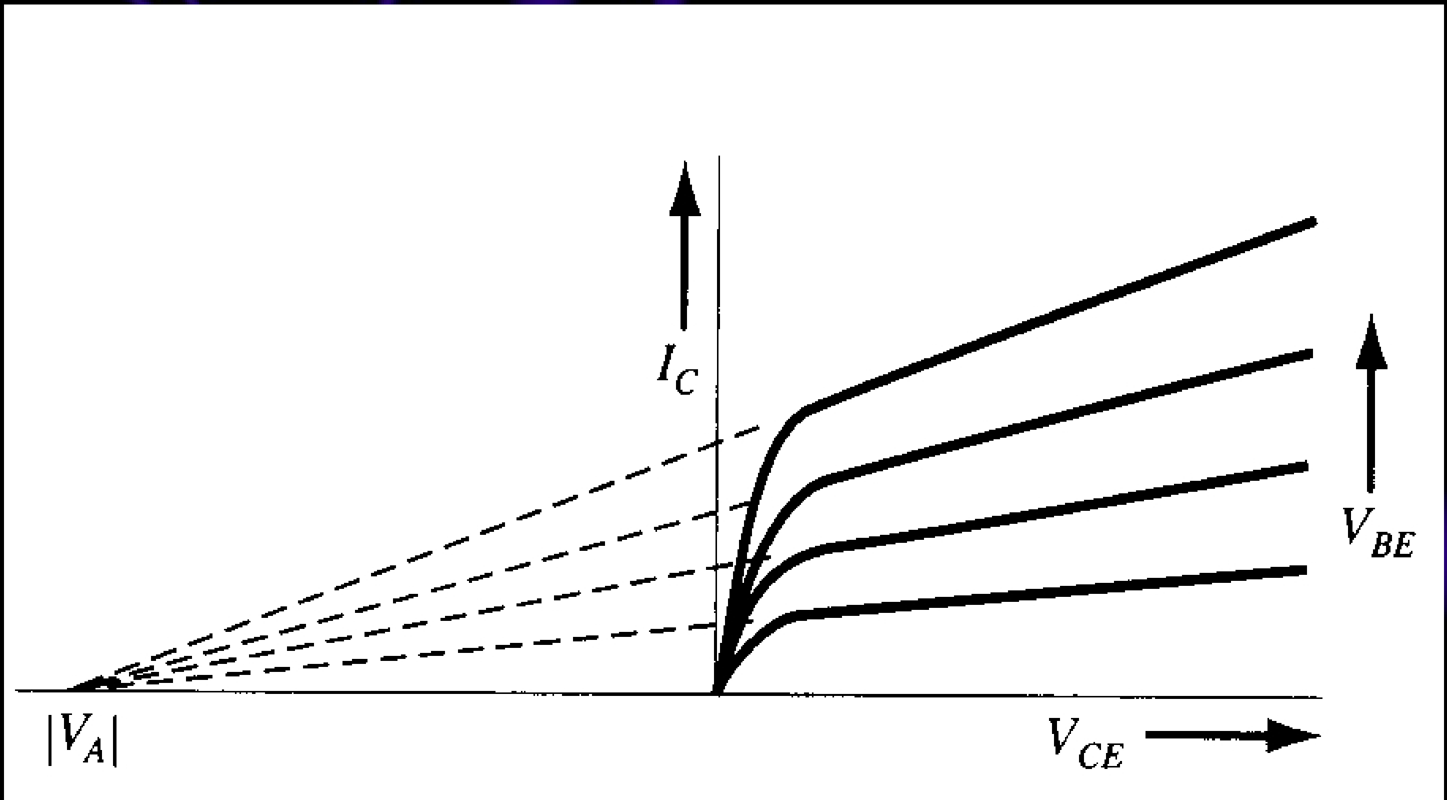
- Increasing V_{BC} (reverse bias), increases the space charge region width
- This reduces x_B
- This will cause an increase in the concentration gradient in the base region, caused by the injection of minority carriers from the emitter
- This will increase the diffusion current through the base
- This results in an increase in the collector current

This is known as the *Early effect*; Early being the First person to identify the problem

The change in the base width and the change in the minority carrier concentration gradient, as the B-C space charge width changes is illustrated below,



The Early effect is illustrated below, where the collector current is plotted against the C-E voltage - the extent to which a structure is influenced by the Early effect is represented by the Early voltage, also shown



Ideally the collector current is independent of the B-C voltage, so the slope in the above graphs would be zero

- The Early effect gives a non-zero slope and leads to a finite output conductance

- If the collector current characteristics are extrapolated to zero collector current, the curves intersect the voltage axis at a point defined as the Early voltage, which is considered to have a positive value

This is an important parameter in transistor design

- Typical values are in the 100-300 volt range
- The actual design used will determine the value, and hence determine the user specifications within a given circuit

We can write that,

$$\frac{dI_C}{dV_{CE}} \equiv g_0 = \frac{I_C}{V_{CE} + V_A}$$

Where V_A and V_{CE} are defined as positive quantities and g_0 is defined as the output conductance. We can now re-write the above equation,

$$I_C = g_0(V_{CE} + V_A)$$

showing explicitly that the collector current is now a function of the C-E voltage or the C-B voltage

Questions

How would you use the doping levels in an npn structure to influence the Early voltage?

In what way would you change them to reduce the Early voltage?

What would be the influence of increasing the base width? What other, perhaps competing, effect would this have?

Example 5

Calculate the change in the neutral base width with a change in the C-B voltage

Consider a uniformly doped npn BJT at 300K

Assume a base doping level of 5 x 10^{16} cm^{-3} and a collector doping level of 2 x 10^{15} cm^{-3}, taking a metallurgical base width of 0.70µm

Calculate the neutral base width as the C-B voltage changes from 2 to 10 volts

The space charge width extending into the base region can be written as,

$$x_{dB} = \left\{ \frac{2\epsilon_s (V_{bi} + V_{CB})}{e} \left[\frac{N_C}{N_B} \cdot \frac{1}{(N_B + N_C)} \right] \right\}^{1/2}$$

or

$$x_{dB} = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_{CB})}{1.6 \times 10^{-19}} \times \left[\frac{2 \times 10^{15}}{5 \times 10^{16}} \cdot \frac{1}{(5 \times 10^{16} + 2 \times 10^{15})} \right] \right\}^{1/2}$$

$$x_{dB} = \{(9.96 \times 10^{-12})(V_{bi} + V_{CB})\}^{1/2}$$

The built-in potential is,

$$V_{bi} = \frac{kT}{e} \ln\left[\frac{N_B N_C}{n_i^2}\right] = 0.718 \text{ volt}$$

For V_{CB} = 2 volts, we find x_{dB} = 0.052μm, and for V_{CB} = 10 volts, we find x_{dB} = 0.103μm

If we neglect the B-E space charge region, which is small as The junction is in forward bias, we can calculate the neutral Base width

For V_{CB} = 2 volts

$$x_B = 0.70 - 0.052 = 0.648\ \mu\text{m}$$

For V_{CB} = 10 volts

$$x_B = 0.70 - 0.103 = 0.597\ \mu\text{m}$$

This example shows that the neutral base width can change by 8% as V_{CB} goes from 2 to 10 volts

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Example 6

Calculate the change in the collector current with a change in the neutral base width, and estimate the Early voltage

Consider a uniformly doped Si npn BJT, with parameters as determined in example 5.

Assume $D_B = 25 cm^2/sec$, $V_{BE} = 0.60V$, and that $x_B << L_B$

The excess minority carrier electron concentration in the base is given by,

$$\delta n_B(x) = \frac{n_{B0}\left\{\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]\sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)\right\}}{\sinh\left(\frac{x_B}{L_B}\right)}$$

If $x_B << L_B$, then $(x_B - x) << L_B$ so we can approximate,

$$\sinh\left(\frac{x_B}{L_B}\right) \cong \left(\frac{x_B}{L_B}\right) \quad \text{and} \quad \sinh\left(\frac{x_B - x}{L_B}\right) \cong \left(\frac{x_B - x}{L_B}\right)$$

The expression for $\delta n_B(x)$ can then be approximated as,

$$\delta n_B(x) \cong \frac{n_{B0}}{x_B}\left\{\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right](x_B - x) - x\right\}$$

and the collector current is now,

$$|J_C| = eD_B\frac{d(\delta n_B(x))}{dx} \cong \frac{eD_B n_{B0}}{x_B}\exp\left(\frac{eV_{BE}}{kT}\right)$$

The value of n_{B0} is calculated as,

$$n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \text{ cm}^{-3}$$

If we let x_B = 0.648μm when V_{CB} = 2V, then,

$$|J_C| = \frac{(1.6 \times 10^{-19})(25)(4.5 \times 10^3)}{0.648 \times 10^{-4}} \exp\left(\frac{0.60}{0.0259}\right) = 3.20 \text{ A/cm}^2$$

with V_{CB} = 10V, this becomes 3.47 A/cm^2

We can write

$$\frac{dJ_C}{dV_{CE}} = \frac{J_C}{V_{CE} + V_A} = \frac{\Delta J_C}{\Delta V_{CE}}$$

Which gives us,

$$\frac{\Delta J_C}{\Delta V_{CE}} = \frac{3.47 - 3.20}{10.6 - 2.6} = \frac{J_C}{V_{CE} + V_A} \cong \frac{3.20}{2.6 + V_A}$$

and the Early voltage is then,

$$V_A \approx 92 \text{ volts}$$

This example indicates how much the collector current can change as the neutral base width changes with a change in the B-C space charge width, and also illustrates the value of a typical Early voltage

High injection

So far we have assumed low injection conditions

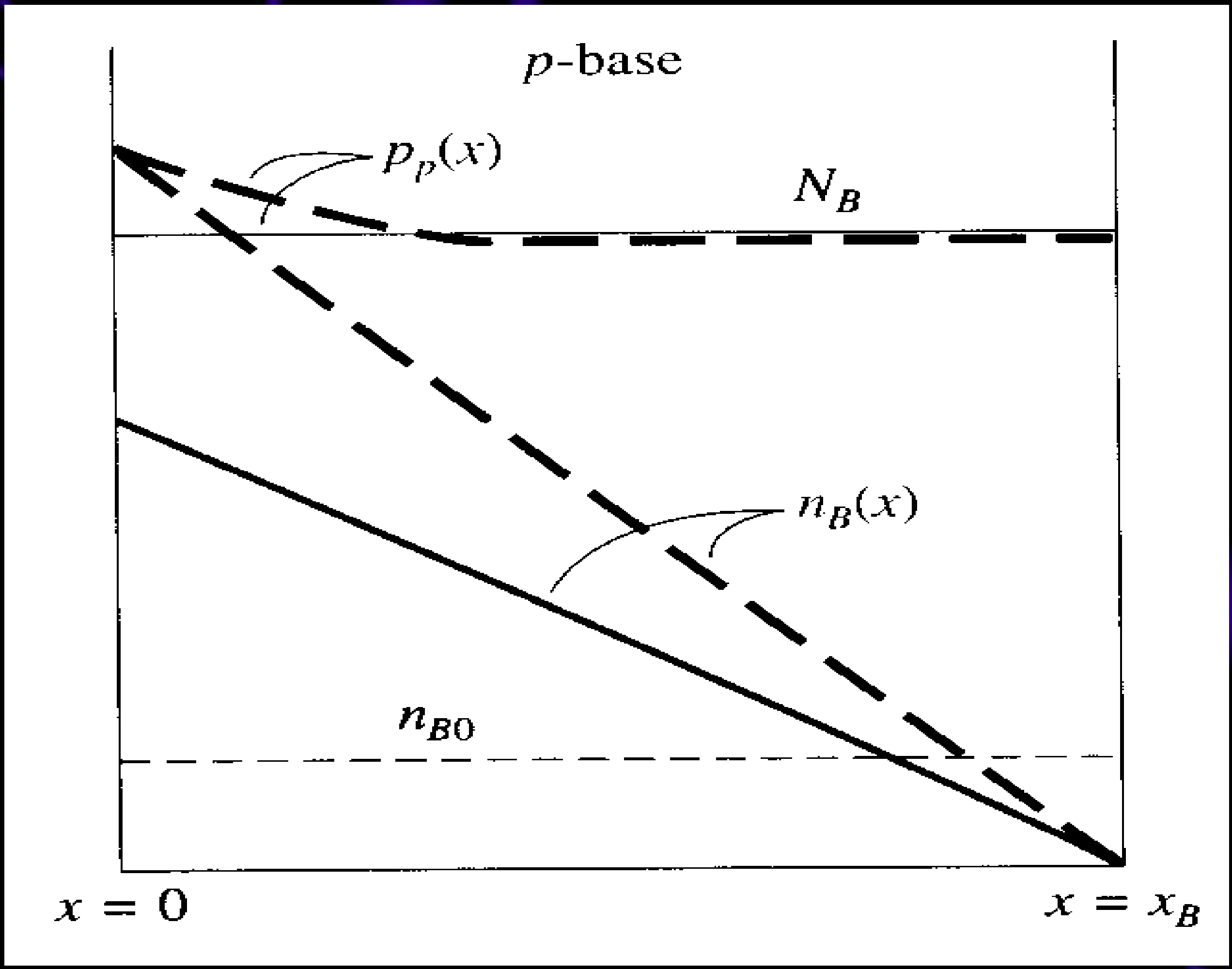
- the minority carrier concentration in the base remains low compared to the majority carrier concentration

As V_{BE} increases, the injected minority carrier concentration may approach or even exceed the majority carrier Concentration

This will cause two things to happen

- Reduction in emitter efficiency
- The collector current will increase at a slower rate as V_{BE} increases, in effect an increased series resistance is created

If we assume quasi-charge neutrality, then the majority carrier hole concentration in the p-base will increase due to the excess holes as shown below,



First effect - reduction in emitter efficiency

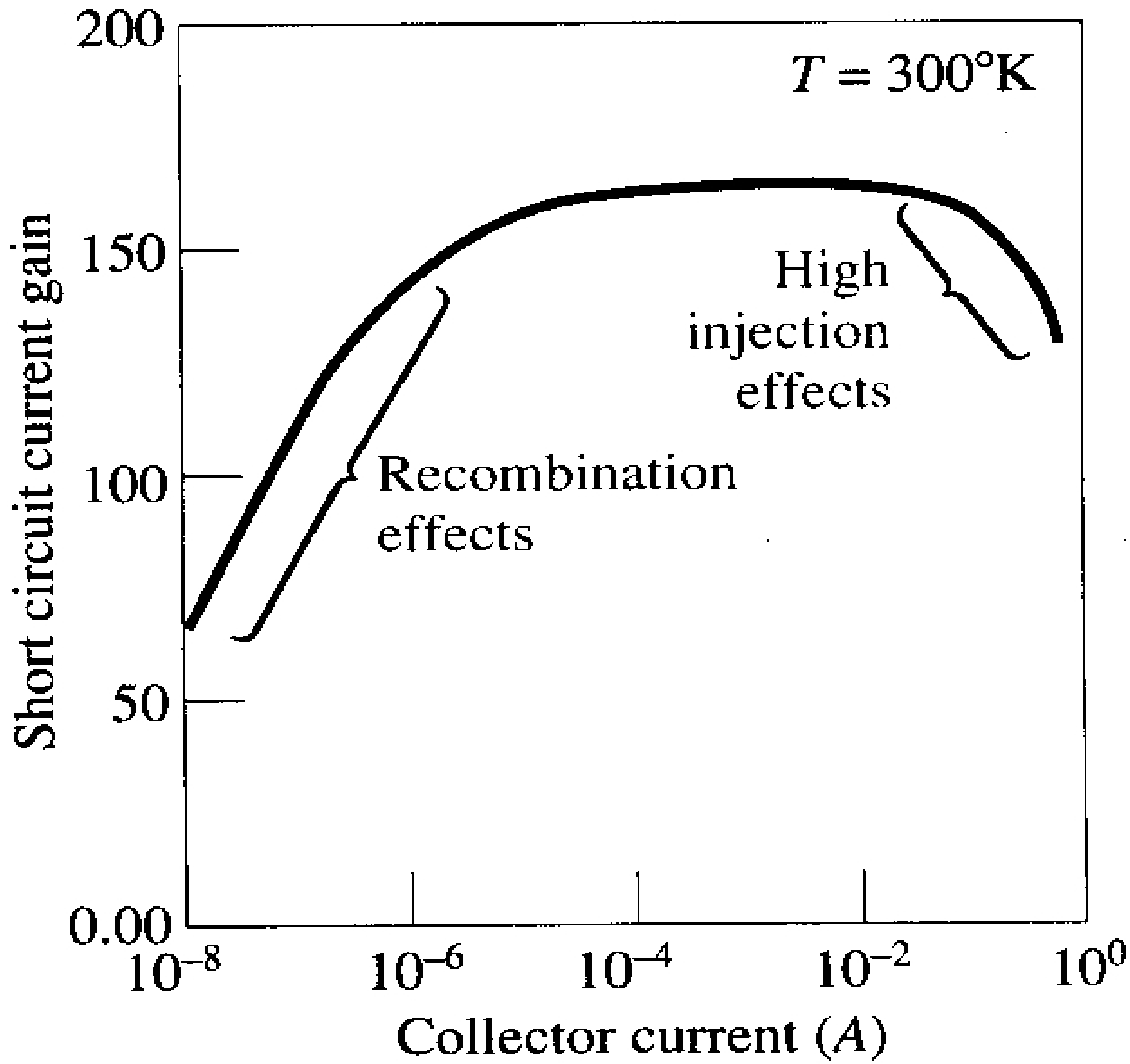
Since the majority carrier hole concentration at x = 0 increases with high injection, more holes are injected back into the emitter due to the forward biased B-e voltage

An increase in hole injection causes an increase in the J_{PE} current and an increase in J_{PE} reduces the emitter injection efficiency

- Hence, the common emitter current gain decreases under conditions of high injection

This is illustrated below, where the common emitter gain For a typical BJT is plotted against collector current

The low gain at low currents is due to the small recombination factor and the drop-off at the high current is due to the high injection effect



200
150
100
50
0.00
Short circuit current gain
T = 300°K
High
injection
effects
Recombination
effects
10−8
10−6
10−4
10−2
100
Collector current (A)

Second effect - slower increase in collector current with V_{BE}

At low injection the majority carrier hole concentration at x =0 for the npn BJT is,

$$p_p(0) = p_{p0} = N_a$$

and the minority carrier electron concentration is,

$$n_p(0) = n_{p0} \exp\left(\frac{eV_{BE}}{kT}\right)$$

The p-n product is,

$$p_p(0)n_p(0) = p_{p0}n_{p0} \exp\left(\frac{eV_{BE}}{kT}\right)$$

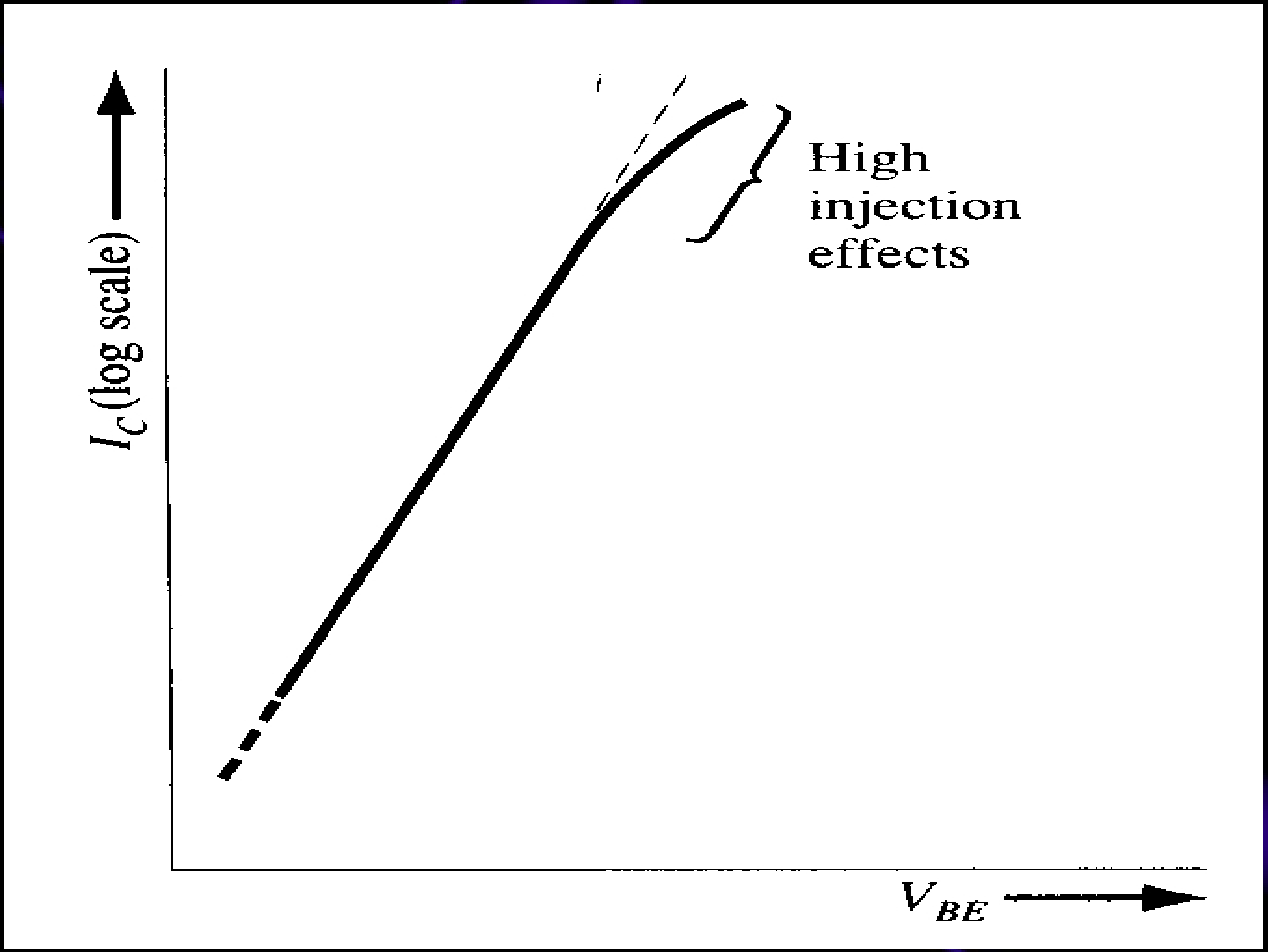
At high injection this last equation still applies. However, $p_P(0)$ will also increase, and for very high injection it will increase at nearly the same rate as $n_P(0)$

This will asymptotically approach the function

$$n_p(0) \approx n_{p0} \exp\left(\frac{eV_{BE}}{2kT}\right)$$

The excess minority carrier concentration in the base, and hence the collector current, will increase at a slower rate with B-E voltage in high injection than low injection

The high injection effect is very similar to the effect of a series resistance in a p-n junction diode

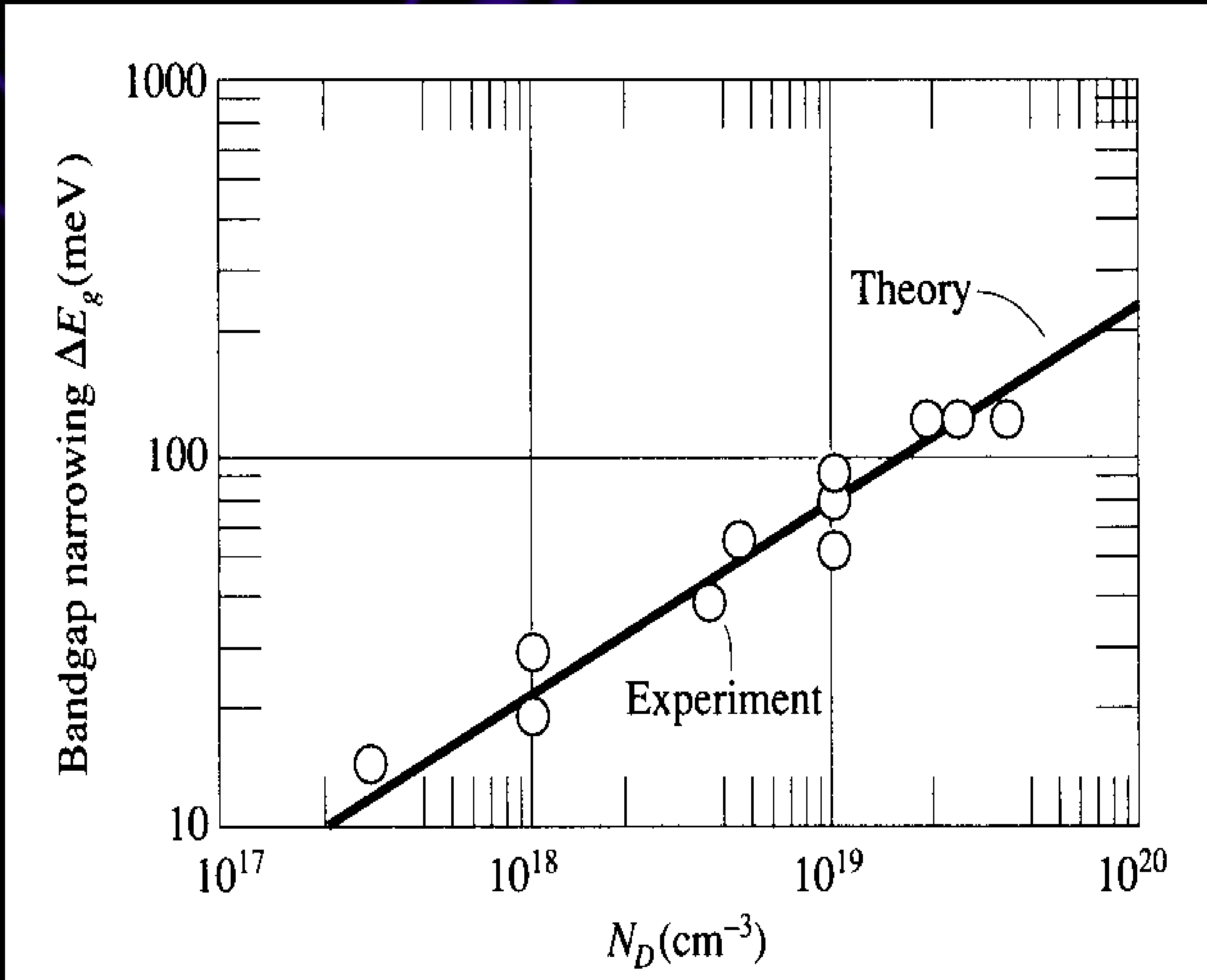


Collector current vs base-emitter voltage showing high injection effects

Emitter bandgap narrowing

As silicon becomes heavily doped the discrete donor energy Level in an n-type emitter splits into a band of energies

- The distance between donor atoms decreases as the concentration increases, and the splitting is caused by the interaction of the donor atoms with each other
- As the doping continues to increase, the donor band widens, becomes skewed, and moves upward towards the conduction band edge, eventually merging with it
- At this point the effective band gap has decreased



1000
100
10
Bandgap narrowing ΔE_g (meV)
Theory
Experiment
10^{17}
10^{18}
10^{19}
10^{20}
N_D(cm^{-3})

A reduction in the bandgap energy increases the intrinsic carrier concentration,

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

In a heavily doped emitter, the intrinsic carrier concentration can be written as,

$$n_{iE}^2 = N_c N_v \exp\left(\frac{-(E_{g0} - \Delta E_g)}{kT}\right) = n_i^2 \exp\left(\frac{\Delta E_g}{kT}\right)$$

where E_{g0} is the bandgap energy at a low doping concentration and ΔE_g is the bandgap narrowing factor

The emitter injection efficiency factor is (as before),

$$\gamma = \frac{1}{1 + \dfrac{p_{E0} D_E L_B}{n_{B0} D_B L_E} \cdot \dfrac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

The term P_{E0} is the thermal-equilibrium minority carrier concentration in the emitter and can be written as,

$$p_{E0} = \frac{n_{iE}^2}{N_E} = \frac{n_i^2}{N_E} \exp\left(\frac{\Delta E_g}{kT}\right)$$

As the emitter doping increases, ΔE_g increases, thus p_{E0} does not continue to decrease with emitter doping - ie emitter injection efficiency begins to fall off rather than increasing with emitter doping

Example 7

Determine the increase in P_{E0} in emitter doping due to bandgap narrowing

Consider a silicon emitter at T=300K, assume the emitter doping increases from 10^{18} to 10^{19} cm^{-3}

Neglecting bandgap narrowing we have, for each respectively,

$$p_{E0} = \frac{n_i^2}{N_E} = \frac{(1.5 \times 10^{10})^2}{10^{18}} = 2.25 \times 10^2 \text{ cm}^{-3}$$

$$p_{E0} = \frac{(1.5 \times 10^{10})^2}{10^{19}} = 2.25 \times 10^1 \text{ cm}^{-3}$$

Taking into account the bandgap narrowing we obtain,

$$p_{E0} = \frac{(1.5 \times 10^{10})^2}{10^{18}} \exp\left(\frac{0.030}{0.0259}\right) = 7.16 \times 10^2 \text{ cm}^{-3}$$

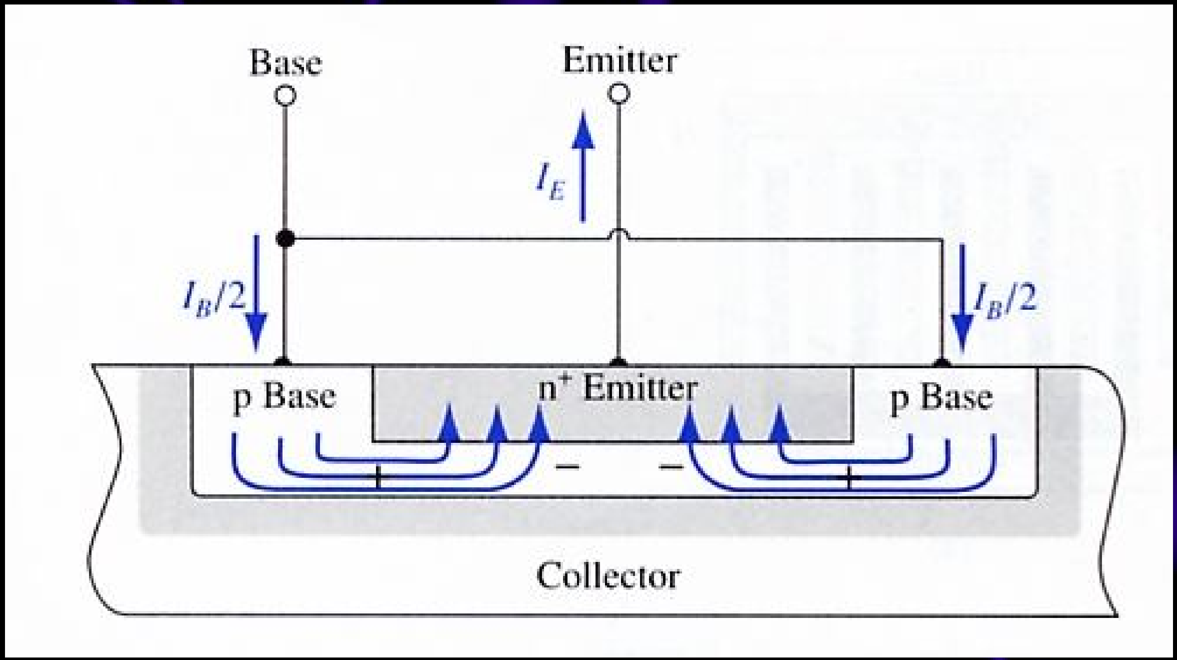
$$p_{E0} = \frac{(1.5 \times 10^{10})^2}{10^{19}} \exp\left(\frac{0.1}{0.0259}\right) = 1.07 \times 10^3 \text{ cm}^{-3}$$

Hence, if the emitter doping increases in this way, the thermal equilibrium minority carrier concentration actually increases by a factor of 1.5, rather than decreasing by the expected factor of 10. This effect is due to bandgap narrowing

This leads the emitter injection efficiency to decrease, and hence the transistor gain <u>decreases</u> compared to the value we expect

Current crowding

The base region is generally <1µm thick, so there can be a significant base resistance



The non-zero base resistance results in a lateral potential difference under the emitter region

For an npn transistor, the potential decreases from the edge Of the emitter toward the centre

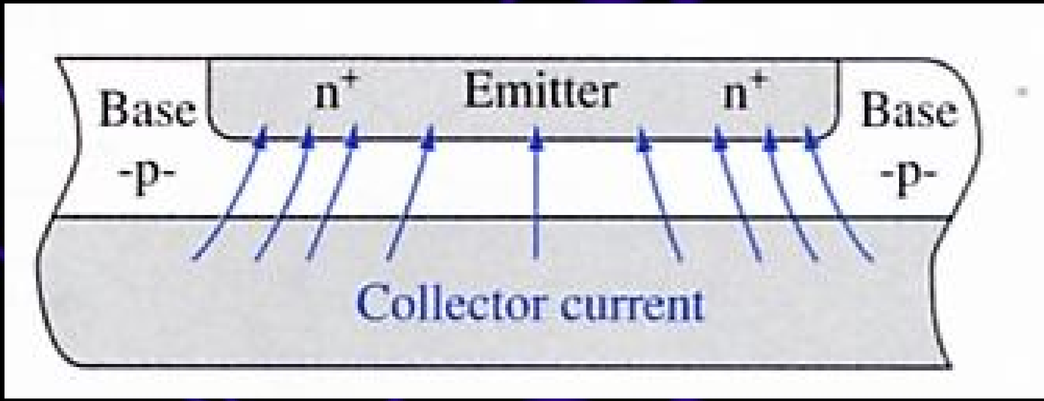
- The emitter is highly doped so to a first approx. can be considered an equipotential region

The number of electrons from the emitter injected into the Base is exponentially dependent on V_{BE}

- With the lateral drop in the base between the edge and centre of the emitter, more electrons will be injected near the emitter edges than the emitter centre

- This causes ‘current crowding’

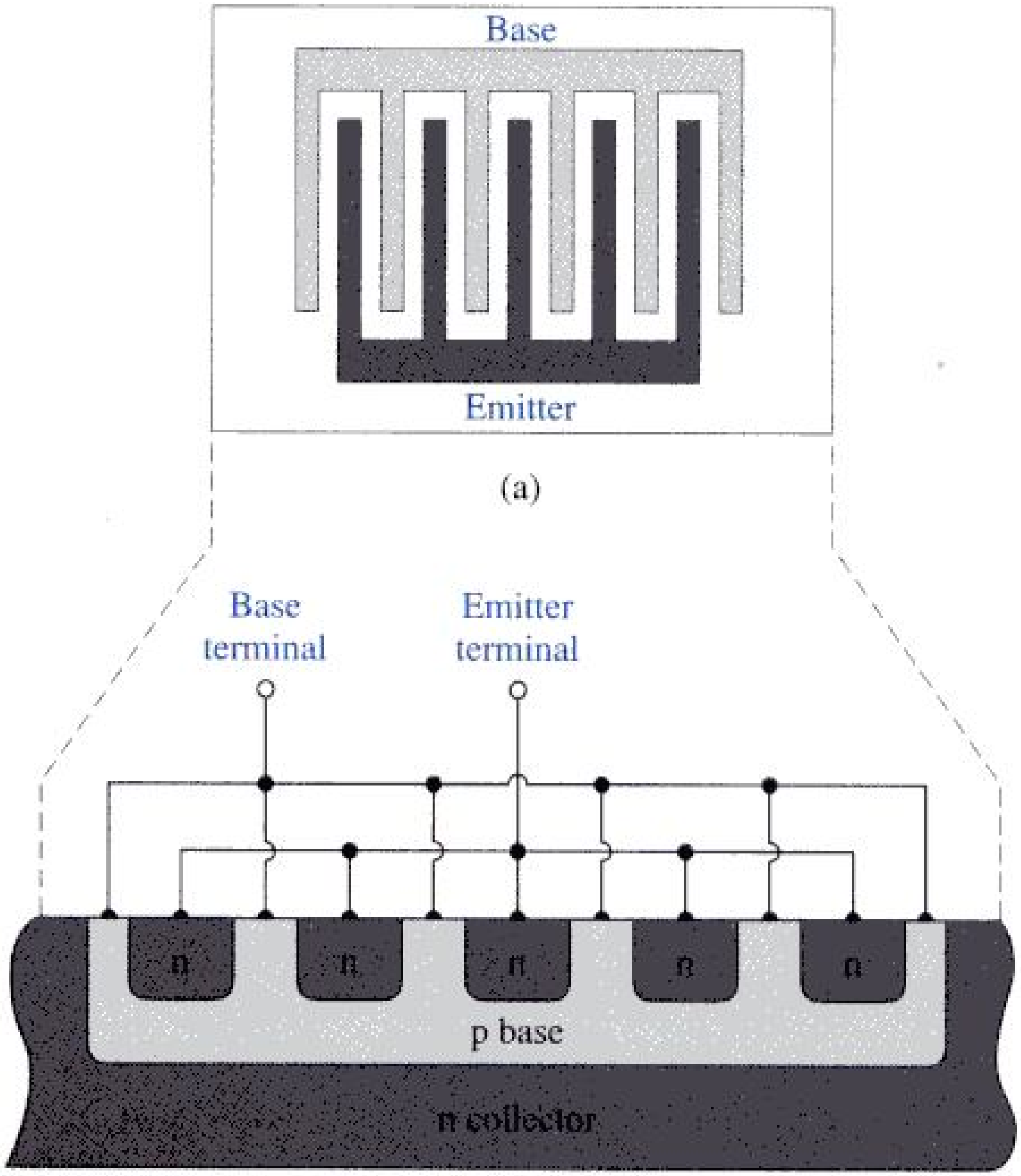
- The larger current density near the emitter edge may cause localised heating and localised high injection effects



The non-uniform emitter current also results in a non-uniform Lateral base current under the emitter

- A 2-D analysis would be required to calculate the actual PD vs distance bacuse of this

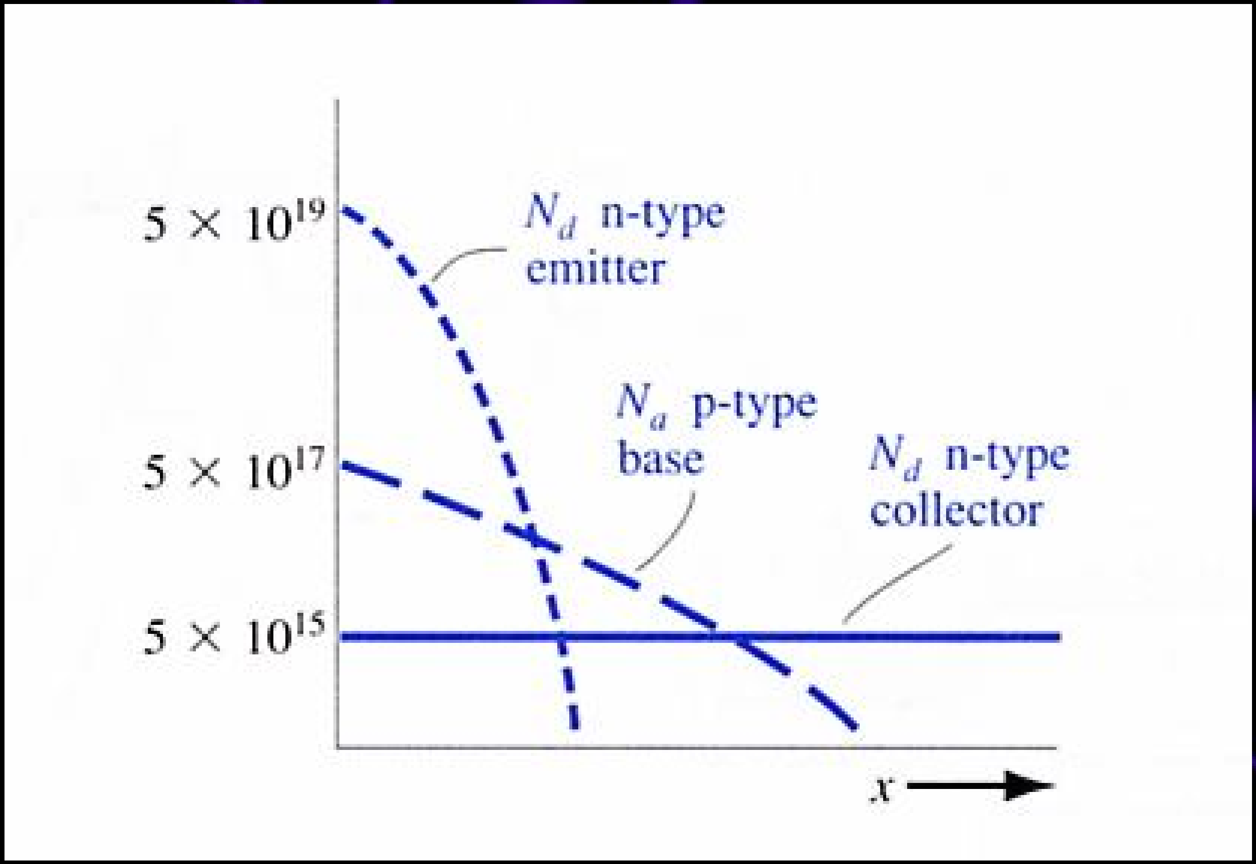
Power transistors require large emitter areas to maintain reasonable current desnsities, to avoid current-crowding these transistors are usually designed with narrow emitter widths and fabricated with an interdigitated design - in effect many narrow emitters are connected in parallel to achieve the required emitter area


Base
Emitter
Base terminal
Emitter terminal
n
n
n
n
n
p base
n collector
(a)
(b)

Non-uniform base doping

We have assumed uniformly doped regions in our BJTs so far

- Technology limitations mean this is rarely the case



Questions

Why are these profiles typical in terms of the doping technology used?

What could we do to make them more linear?

A graded impurity concentration leads to an induced electric field. For a p-type base region we can write,

$$J_p = e\mu_p N_a \mathrm{E} - eD_p \frac{dN_a}{dx} = 0$$

Then,

$$E = +\left(\frac{kT}{e}\right)\frac{1}{N_a}\frac{dN_a}{dx}$$

Electrons are injected from the n-type emitter into the base and the minority carrier base electrons begin diffusing towards The collector region

- The induced electric field in the base, due to non-uniform doping, produces a force on the electrons in the direction toward the collector

- This drift current is thus an aid to the existing diffusion current, although the total current across the base will remain constant

The induced electric field due to non-uniform doping will alter the minority carrier distribution throughout the base so that the sum of the drift and diffusion currents become a constant

- Uniform base doping theory remains useful in estimating base characteristics
- The principal effect is a reduction in the likelihood of device breakdown through ‘punch-through’

Breakdown Voltage

Two mechanisms must be considered

- Punch-through
- Avalanche breakdown

Punch-through

- As the reverse-bias B-C voltage increases, the depletion region encroaches further into the base and can eventually occupy all of it
- This will cause a large surge in collector current and loss of transistor action

Neglecting the contribution to base narrowing from the forward biased E-B junction, punch-through will occur when $x_{dB} = W_B$ where,

$$x_{dB} = W_B = \left\{ \frac{2\epsilon_s (V_{bi} + V_{pt})}{e} \cdot \frac{N_C}{N_B} \cdot \frac{1}{N_C + N_B} \right\}^{1/2}$$

where V_{pt} is the reverse-biased B-C voltage at punch-through negelecting V_{bi} compared to V_{pt}, we can solve V_{pt} as,

$$V_{pt} = \frac{eW_B^2}{2\epsilon_s} \cdot \frac{N_B(N_C + N_B)}{N_C}$$

Example 8

Design the collector doping and collector width of a BJT to meet a punch-through voltage specification

Consider a uniformly doped Si BJT with a metallurgical base width of 0.5μm, and a base doping level of $N_B = 10^{16}\ cm^{-3}$. The punch-through voltage is to be $V_{pt} = 25V$.

The maximum collector doping concentration can be determined as,

$$25 = \frac{(1.6 \times 10^{-19})(0.5 \times 10^{-4})^2(10^{16})(N_C + 10^{16})}{2(11.7)(8.85 \times 10^{-14})N_C}$$

$$12.94 = 1 + \frac{10^{16}}{N_C}$$

$$N_C = 8.38 \times 10^{14}\ \mathrm{cm}^{-3}$$

We can then determine,

$$x_n = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left(\frac{N_B}{N_C} \cdot \frac{1}{N_B + N_C} \right) \right\}^{1/2}$$

Neglecting V_{bi} compared to $V_R = V_{pt}$, we obtain

$$x_n = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(25)}{1.6 \times 10^{-19}} \left(\frac{10^{16}}{8.38 \times 10^{14}} \right) \left(\frac{1}{10^{16} + 8.38 \times 10^{14}} \right) \right\}^{1/2}$$

or

$$x_n = 5.97_{\mu\mathrm{m}}$$

Avalanche breakdown - tends to occur at much higher potentials than punch-through for most BJT designs, so is less important

Frequency limitations

We need to consider two effects

- Time-delay factors
- Transistor cut-off frequency

The BJT is a transit time device. When a voltage across the B-E junction increases, additional carriers are created in the base, they must then diffuse and be collected in the collector Region

As the frequency increases the transit time will become appreciable compared to the period of the input signal

- Output will no longer be in phase with the input and the current gain will decrease

The total emitter-to-collector constant or time delay can be written as,

$$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$$

τ_{ec} = Emitter-to-collector time delay

τ_e = Emitter-base junction capacitance charging time

τ_b = Base transit time

τ_d = Collector depletion region transit time

τ_c = Collector capacitance charging time

The common-base current gain as a function of frequency can be written as,

$$\alpha = \frac{\alpha_0}{1 + j\dfrac{f}{f_\alpha}}$$

Where α_0 is the low frequency common base current gain and f_α is defined as the alpha cutoff frequency, which is related to the emitter to collector delay τ_{EC} as

$$f_\alpha = \frac{1}{2\pi\tau_{ec}}$$

When the frequency is equal to the alpha cutoff frequency, the magnitude of the common-base current gain is $1/\sqrt{2}$ of it low frequency value

We can relate the alpha cutoff frequency to the common emitter current gain by considering,

$$\beta = \frac{\alpha}{1-\alpha}$$

$$|\beta| = \left|\frac{\alpha}{1-\alpha}\right| \cong \frac{f_\alpha}{f}$$

$$\boxed{f_T = \frac{1}{2\pi\tau_{ec}}}$$

Where we have assumed $\alpha_0 \sim 1$. When the signal frequency is equal to f_α, the magnitude of the common emitter current gain is equal to 1, defined as the cutoff frequency, f_T

We can also write the common emitter current gain as,

$$\beta = \frac{\beta_0}{1 + j(f/f_\beta)}$$

where f_β is called the beta cutoff frequency

Combining these equations gives,

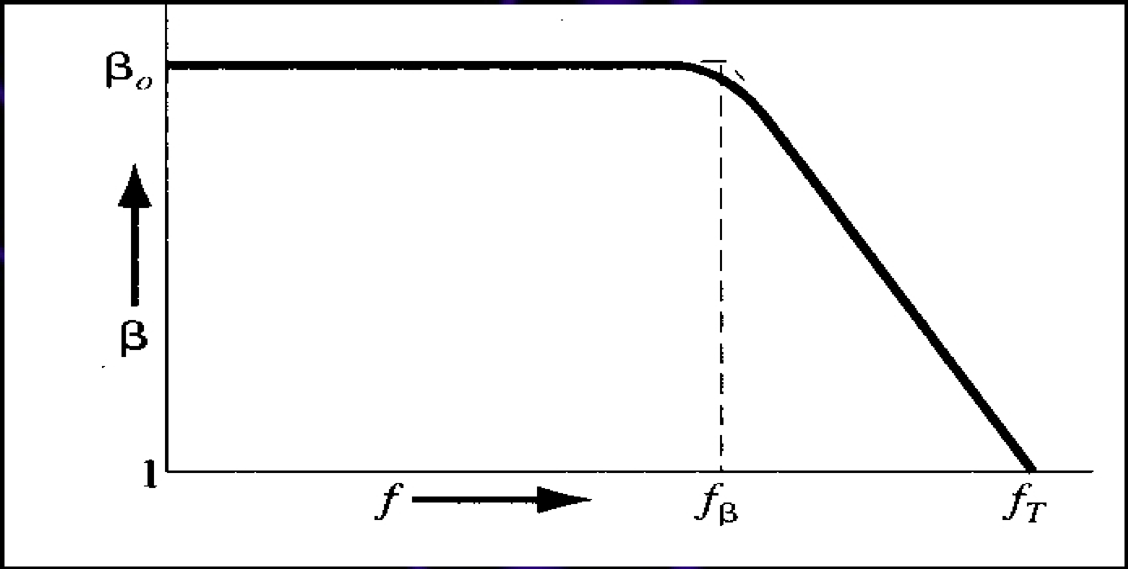
$$\beta = \frac{\alpha}{1-\alpha} = \frac{\dfrac{\alpha_0}{1 + j(f/f_T)}}{1 - \dfrac{\alpha_0}{1 + j(f/f_T)}} = \frac{\alpha_0}{1 - \alpha_0 + j(f/f_T)}$$

$$\beta = \frac{\alpha_0}{(1-\alpha_0)\left[1 + j\dfrac{f}{(1-\alpha_0)\,f_T}\right]} \cong \frac{\beta_0}{1 + j\dfrac{\beta_0 f}{f_T}}$$

$$\beta_0 = \frac{\alpha_0}{1-\alpha_0} \cong \frac{1}{1-\alpha_0}$$

and hence the beta cutoff frequency is related to the cutoff frequency by,

$$\boxed{f_\beta \cong \frac{f_T}{\beta_0}}$$



This is a Bode plot of the common emitter current gain as a function of frequency

- Note the log scale, so f_β and f_T usually have very different values

Example 12

Calculate the emitter-to-collector transit time and the cutoff frequency of a silicon BJT at 300K given,

$I_E = 1 \text{ mA}$	$C_{je} = 1 \text{ pF}$
$x_B = 0.5\ \mu\text{m}$	$D_n = 25 \text{ cm}^2/\text{sec}$
$x_{dc} = 2.4\ \mu\text{m}$	$r_c = 20\ \Omega$
$C_\mu = 0.1 \text{ pF}$	$C_s = 0.1 \text{ pF}$

Initially calculate the various time-delay factors. Neglecting parasitic capacitance, the emitter-base junction charging time is,

$$\tau_e = r'_e C_{je}$$

where

$$r'_e = \frac{kT}{e} \cdot \frac{1}{I_E} = \frac{0.0259}{1 \times 10^{-3}} = 25.9 \text{ ohms}$$

$$\tau_e = (25.9)(10^{-12}) = 25.9 \text{ psec}$$

The base transit time is

$$\tau_b = \frac{x_B^2}{2D_n} = \frac{(0.5 \times 10^{-4})^2}{2(25)} = 50 \text{ psec}$$

The collector depletion region transit time is,

$$\tau_b = \frac{x_{dc}}{v_s} = \frac{2.4 \times 10^{-4}}{10^7} = 24 \ \text{psec}$$

The collector capacitance charging time is

$$\tau_c = \tau_c (C_\mu + C_s) = (20)(0.2 \times 10^{-12}) = 4 \ \text{psec}$$

The total emitter-to-collector time delay is then

$$\tau_{ec} = 25.9 + 50 + 24 + 4 = 103.9 \ \text{psec}$$

so that the cutoff frequency is calculated as

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi(103.9 \times 10^{-12})} = 1.53 \text{ GHz}$$

If we assume a low-frequency common emitter current gain of β = 100, then the beta cutoff frequency is

$$f_\beta = \frac{f_T}{\beta_0} = \frac{1.53 \times 10^9}{100} = 15.3 \text{ MHz}$$

We can conclude that high-frequency BJTs need small device geometries in order to reduce capacitances, and narrow base widths in order to reduce the base transit time

Revision problems

For a uniformly doped n^+pn BJT in thermal equilibrium

(a) Sketch the energy band-diagram

(b) Sketch the electric field through the device

(c) Repeat parts (a) and (b) for the transistor in the forward active region

A uniformly doped silicon npn BJT is to be biased in the Forward-active mode with the B-C junction reverse biased At 3 volts. The metallurgical base width is 1.10µm. The transistor doping levels are $N_E = 10^{17}$, $N_B = 10^{16}$ and $N_C = 10^{15}$ cm^{-3}.

(a) Calculate the B-E voltage at which the minority carrier electron concentration at x=0 is 10% of the majority hole concentration

(b) At this bias, determine the minority carrier hole concentration at x'=0

(c) Determine the neutral base width for this bias