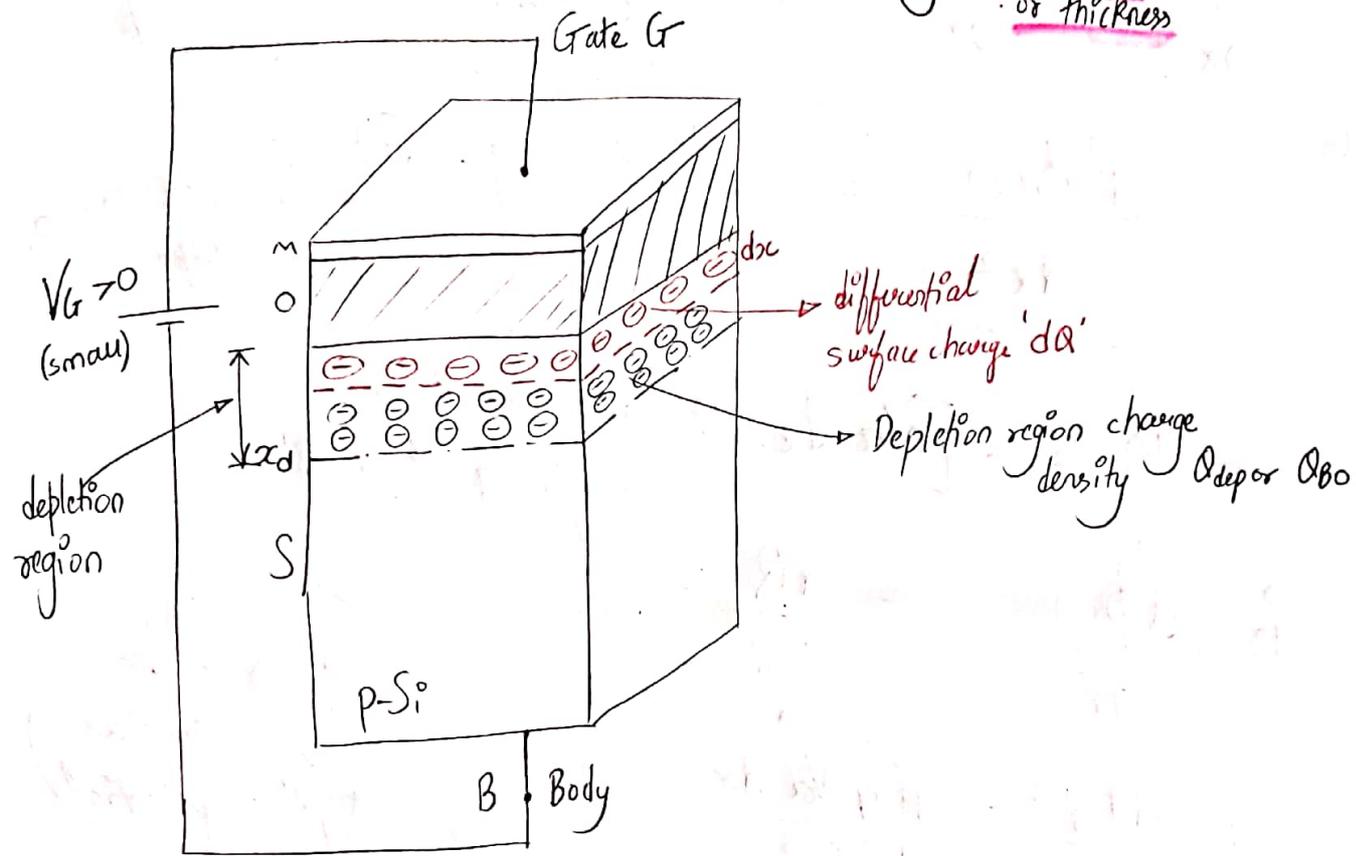


↳ Derivation for x_d (depletion region depth) :
or thickness

01
19/1/2020



- Consider an elementary surface having thickness dx & having differential surface charge ' dQ '
- We assume here that the semiconductor is uniformly doped, i.e. N_A is constant.
- The change in surface potential required to displace this surface charge sheet ' dQ ' by a distance ' dx ' away from the surface can be found by solving Poisson's equation,

Statement
①

$E = - \frac{\partial \psi}{\partial x}$ (E-field) ; $\frac{\partial E}{\partial x} = - \frac{\rho}{\epsilon}$ (space-charge / Gauss's law)

$\frac{d^2 \phi(x)}{dx^2} = - \frac{\rho(x)}{\epsilon_s}$ — (1) — Poisson's equation.

$\rightarrow dQ = -qNa dx$ (From statement 1) — Integrating eqⁿ (1)

$\text{ie } \frac{d^2 \phi(x)}{dx^2} = - \frac{dQ}{\epsilon_s}$

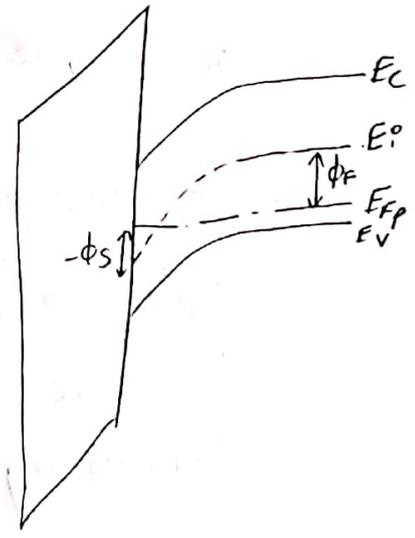
$\text{ie } \frac{d\phi(x)}{dx} = \int_0^x \frac{qNa dx}{\epsilon_s}$

$\phi(x) \rightarrow \phi_s$ surface potential
 function of distance.

$\frac{d\phi(x)}{dx} = \frac{qNa x}{\epsilon_s}$ — (2)

Integrating (2) along the vertical dimensions (perpendicular to the surface) yields,

$\int_{\phi(x)=\phi_F}^{\phi_s} \frac{d\phi(x)}{dx} = \int_{x=0}^{x=x_d} \frac{qNa x}{\epsilon_s}$



{ Integrating limits change from bulk fermi potential to surface fermi potential }

$$\int_{\phi(x)=\phi_F}^{\phi_S} d\phi(x) = \frac{qNa}{\epsilon_s} \int_{x=0}^{x_d} x dx$$

03

$$\phi_S - \phi_F = \frac{qNa}{\epsilon_s} \frac{x_d^2}{2}$$

$$\text{i.e. } \phi_S - \phi_F = \frac{qNa x_d^2}{2\epsilon_s}$$

Thus, the depth of the depletion region is,

$$x_d = \sqrt{\frac{2\epsilon_s |\phi_S - \phi_F|}{qNa}}$$

Unit: cm

— (3)

The above equation shows that the depletion region thickness (x_d) is a function of surface potential $|\phi_S - \phi_F|$

In depletion, the total charge in the semiconductor is just the depletion charge Q_{dep} or Q_{BO} given by,

$$Q_{dep} = Q_{BO} = -qNa x_d$$

$$Q_{BO} = -qNa \sqrt{\frac{2\epsilon_s |\phi_S - \phi_F|}{qNa}}$$

$$Q_{B0} = - \sqrt{2q \epsilon_s N_a |\phi_s - \phi_F|} \quad \text{Unit: } C/cm^2 \quad 04$$

Depletion region charge density (consisting of solely fixed acceptor ions)

→ The amount of this depletion region charge plays a very important role in the analysis of Threshold voltage.

where, $\epsilon_s = \epsilon_{si} \epsilon_0$

\downarrow \swarrow
 11.7 8.854×10^{-14}
 (dielectric constant for Si)

• At inversion, $\phi_s = -\phi_F$

surface fermi potential \rightarrow bulk fermi potential.

Inversion:

• The surface is said to be inverted when the density of mobile electrons on the surface becomes equal to the density of holes in the bulk (p-type) substrate.

Beyond inversion

→ Once the surface is inverted, any further increase in the gate voltage leads to an increase of mobile electron concentration on the surface, but not to an increase of the depletion depth.

→ Thus, the depletion region depth achieved at the onset of surface inversion is also equal to the maximum depletion depth x_{dm} , which remains constant for higher gate voltages.

• Using inversion condition; $\phi_s = -\phi_F$ in eqⁿ (3), we get

$$x_{dm} = \sqrt{\frac{2 \epsilon_s |2\phi_F|}{q N_A}}$$

Unit: cm

→ Maximum depletion region depth at the onset of surface inversion.

∴ The charge in the depletion region will not exceed

$$Q_{Bo\max} = - \sqrt{2q N_A \epsilon_s |2\phi_F|} \text{ C/cm}^2$$

at surface inversion
($\phi_s = -\phi_F$)

* C-V characteristics of MOSCAP

• Study of C-V characteristics of MOSCAP gives a great deal of information about the MOS device and oxide-semiconductor interface.

→ Ideal C-V curve:

• We consider small-signal capacitance given by

$$C = \left| \frac{dQ}{dV} \right| \text{ ie } \left[\frac{\text{differential change in charge}}{\text{differential change in voltage}} \right]$$

• Assume that there are no trapped charges in oxide and oxide-semiconductor interface.

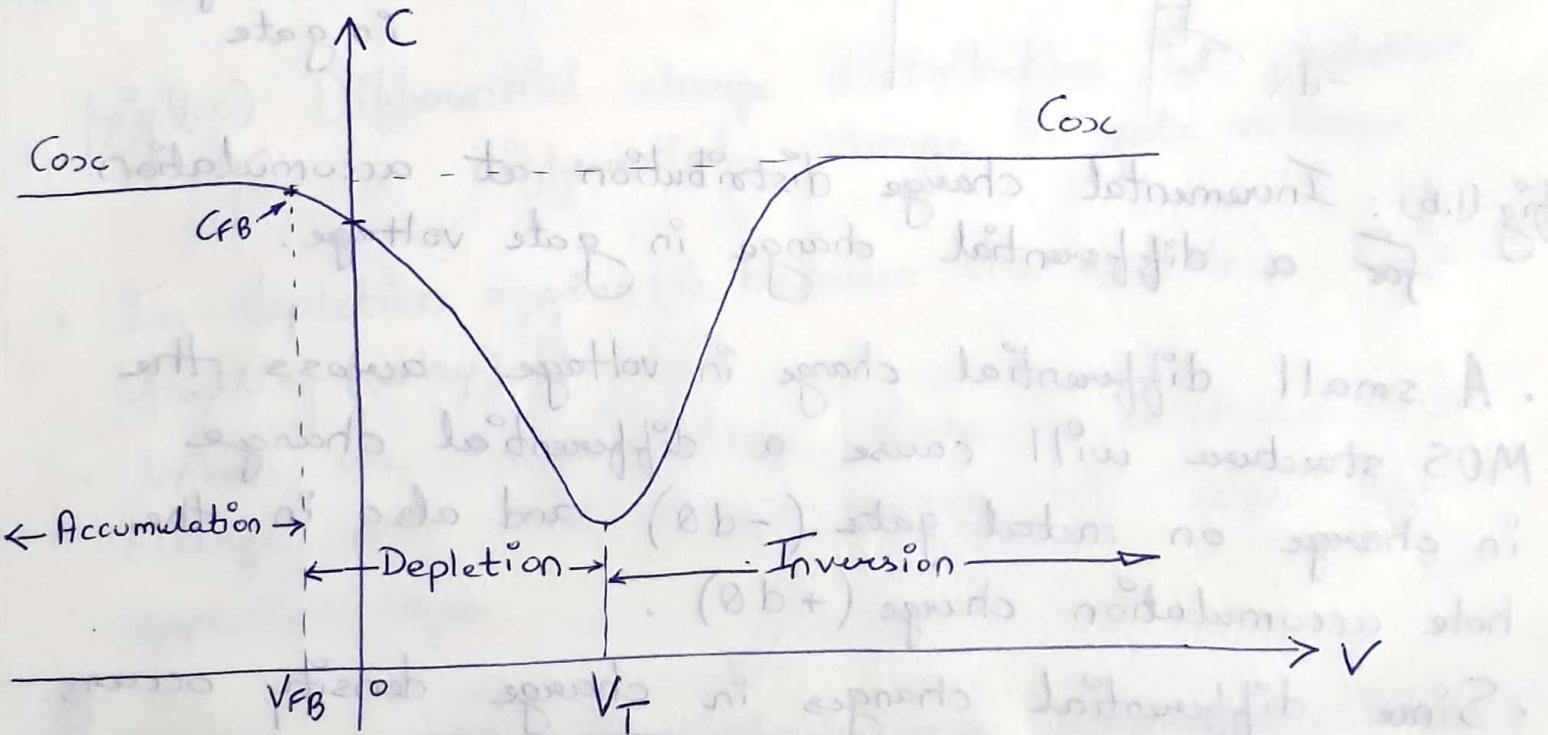


fig (1.a): Ideal low-frequency C-V curve for a MOSCAP with ptype substrate

V_{FB} → Flat-band voltage.

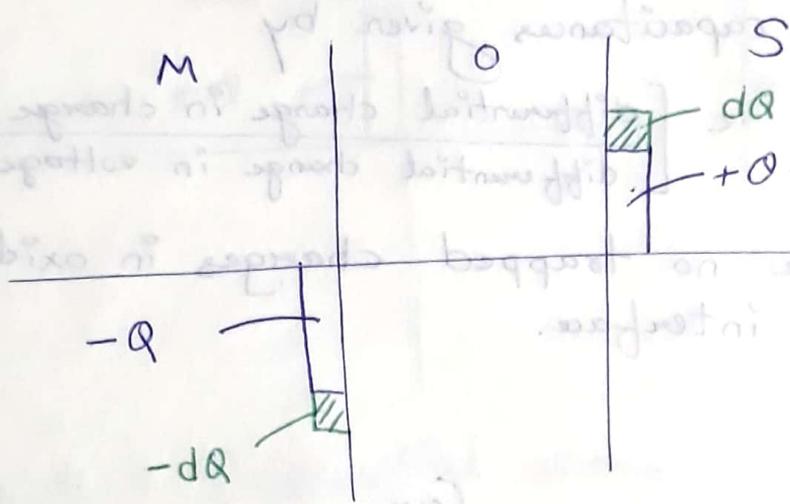
V_T → Threshold voltage.

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3 operating conditions of interest in MOSCAP are,
a) Accumulation b) Depletion c) Inversion

1) Accumulation:

When a negative gate voltage is applied, it induces accumulation of holes beneath Si-SiO₂ interface.



Accumulation Case

+Q ⇒ +ve charge in semiconductor.
-Q ⇒ -ve charge induced in gate

Fig (1.b): Incremental charge distribution at accumulation for a differential change in gate voltage.

A small differential change in voltage across the MOS structure will cause a differential change in charge on metal gate (-dQ) and also in the hole accumulation charge (+dQ).

Since differential changes in charge density occurs at the edges of oxide

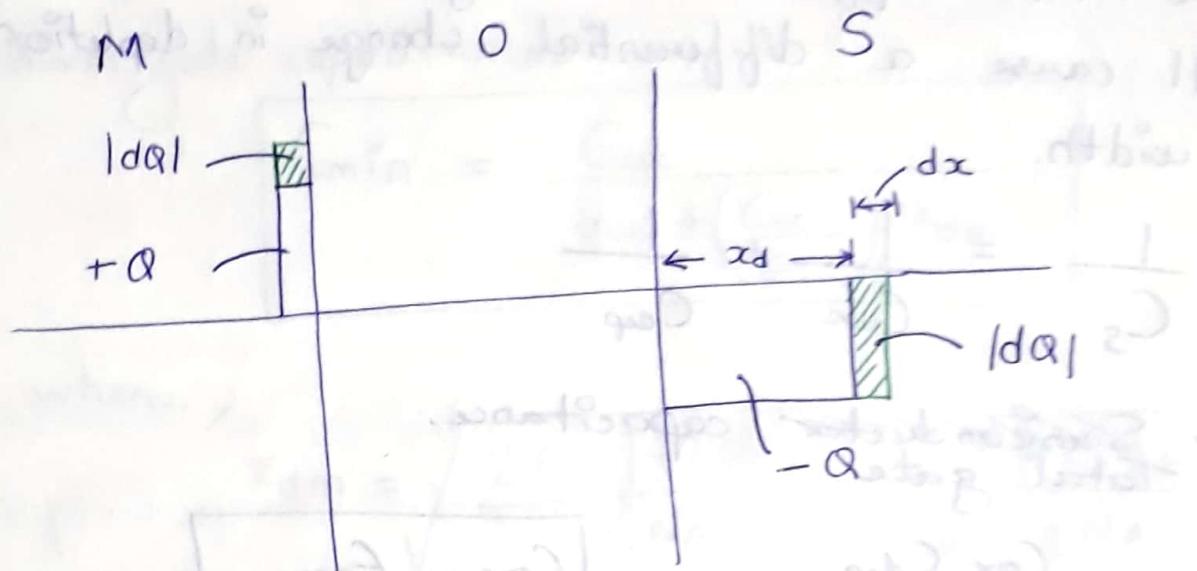
$$C_{(acc)} = C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

Thus capacitance at accumulation is just oxide capacitance.

Similar to a Parallel Plate capacitor, in this case we have a +ve charge in semiconductor and a -ve charge in gate, separated by a oxide.

2) Depletion:

When a small +ve gate voltage is applied to a MOSCAP, it induces a depletion region in semiconductor.



big (l.c) Differential charge distribution at depletion for a differential change in gate voltage.

- In depletion region (ie between V_{FB} and V_T), C is falling (why)?
- When we change voltage, there is a change in charge and that change occurs at 'edge' of depletion layer.

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Extra!!

- (These changes are incremental changes)
- Change in increment +ve charge (dQ) at gate
 - Change in increment -ve charge (dQ) in semiconductor at edge of depletion layer.

- In depletion case, there is oxide capacitance and depletion layer capacitance (which are in series).
- Thus, a small differential change in voltage across C will cause a differential change in depletion layer width.

ie $\frac{1}{C_s} = \frac{1}{C_{ox}} + \frac{1}{C_{dep}}$

$C_s \rightarrow$ ~~Semiconductor~~ Total gate capacitance.

$C_s = \frac{C_{ox} C_{dep}}{C_{ox} + C_{dep}}$ $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$

$C_{dep} = \frac{\epsilon_s}{x_d}$

$x_d \rightarrow$ depletion layer width.

ie $C_s = \frac{C_{ox}}{1 + \frac{C_{ox}}{C_{dep}}} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s}\right) x_d}$

\therefore As $x_d \uparrow$ ser, total $C_s \downarrow$ ser.

This explains why Capacitance goes on falling between V_{FB} and V_T (ie depletion case) because distance between incremental charges are changing as show in fig(1.c).

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Extra!!

At maximum depletion width (x_d)

↓
Minimum gate capacitance's

$$C_{\min} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s}\right) x_{dm}}$$

where,

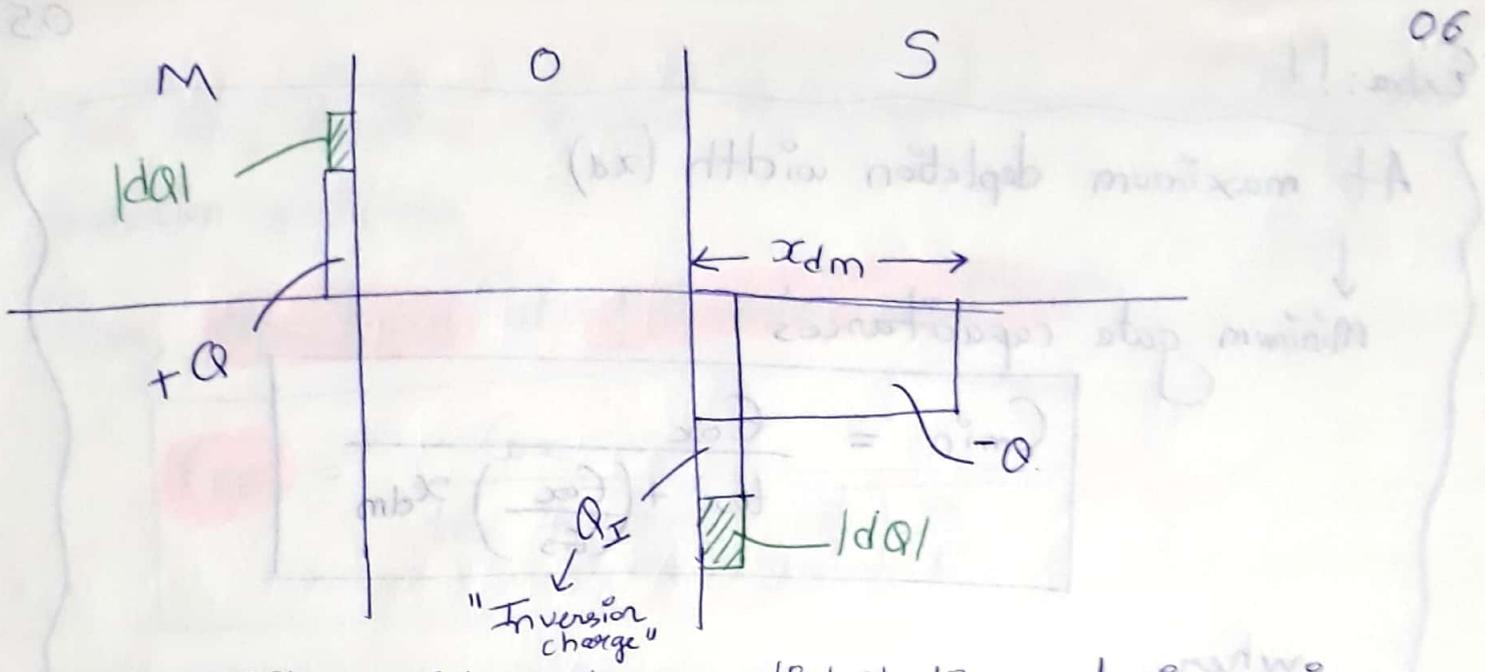
$$x_{dm} = \sqrt{\frac{2\epsilon_s | -2\phi_f |}{qNA}} = \sqrt{\frac{4\epsilon_s \phi_f}{qNA}}$$

3) Inversion:

- In ideal case, a small incremental change in gate voltage will cause a differential change in inversion layer charge density
- Space-charge width (x_d) does not rise beyond V_T , thus inversion charge corresponds to change in voltage. Thus capacitance is again just oxide capacitance ' C_{ox} '.

$$C_{(Inv)} = C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

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fig(1.d): Differential charge distribution at inversion for a low-frequency differential change in gate voltage

In inversion region, C again rises up to C_{ox} ?

- This means, "incremental charges" are now coming from $Si-SiO_2$ interface. (As depletion layer width reaches max value x_{dm})
- { Beyond V_T , we get more mobile charges appear near $Si-SiO_2$ interface }
- In case of inversion, the incremental charges are coming from mobile -ve charges near the interface. as shown in fig.(1.d).

That explains qualitatively behavior of C as a function of gate voltage at low frequency.

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* Flat band condition occurs between accumulation and depletion conditions: 07

Thus, capacitance at flat-band condition is,

$$C_{FB} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s}\right) \sqrt{\left(\frac{kT}{q}\right) \frac{\epsilon_s}{qN_a}}}$$

* Frequency effects on a MOSCAP C-V curve :

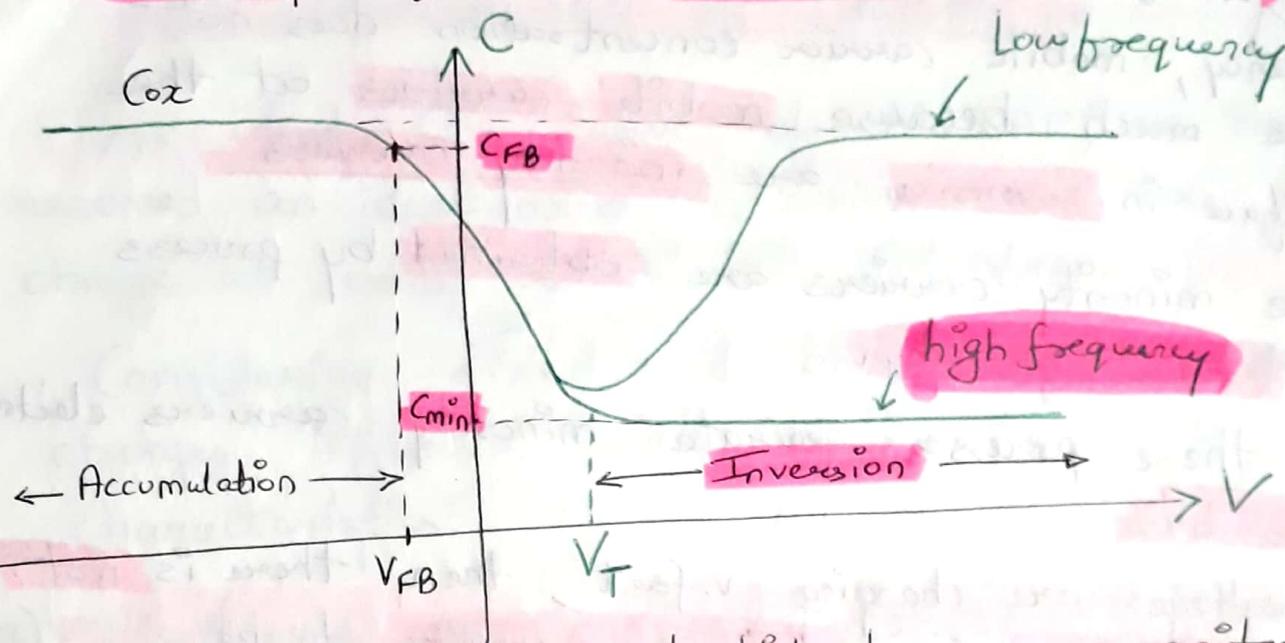


fig (2.a) : Low-frequency and high-frequency capacitance versus gate voltage of a MOSCAP with a p-type substrate

- When a MOSCAP is biased in inversion condition, and if frequency is high, then we cannot provide changes in the mobile carrier concentration (ie electrons) at the interface very easily.
- At high frequency, voltage changes rapidly, thus there is no way Q_I (inversion charge) will be able to respond in time ie the electrons have no opportunity to respond.
- Thus, at high frequency, only depletion region differential charge changes, thus capacitance almost remains constant at C_{min} . as shown in fig (2.a).

- The argument that in inversion condition at high frequency, mobile carrier concentration does not change much because mobile carriers at the interface in inversion are minority carriers.
- These minority carriers are obtained by process of thermal generation.
- But these processes generate minority carriers electrons takes time.
- If voltages are changing v. fast, then there is not sufficient time for minority carriers to be generated.
- Therefore, for rapid changes in gate voltage, in inversion case, the ^{differential} change in charges cannot come from inversion charge (Q_i), it comes in fact from edge of depletion layer charge (Q_D).
- Because, majority carrier charge can be changed quickly in response to change in voltage.
- Thus, at high frequency, incremental charges are coming from depletion charge (since width of depletion layer saturates) beyond V_T , capacitance in inversion almost does not change much and remains constant at C_{min} .

That explain's C behavior as a function of voltage at high frequency.

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