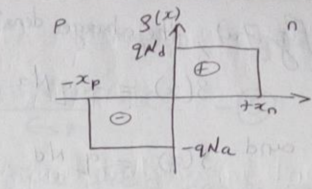
- figure to the right shows volume space charge density distribution in the pn junction (Assuming uniform doping ; abrupt junctn approximation)



- The space - charge region extends from $x = -x_p$ in the p-region to $x = x_n$ in the n-region

Expression for Electric field (pn Junction for zero applied bias):

- The electric field is determined from Poisson's equation (For one-dimensional analysis) :-

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon_{si}} = -\frac{dE(x)}{dx} \quad \text{— ①}$$

where, $\phi(x)$ is the electrical potential (position-dependent potential)
$E(x)$ is the electric field (position-dependent electric field).
$\rho(x)$ is position-dependent space-charge density.
ϵ_{si} is the permittivity of the semiconductor.

Gauss's law :- $\frac{\partial E}{\partial x} = \frac{\rho}{\epsilon}$ — (space charge)

Electric flux: $E = -\frac{\partial \phi}{\partial x}$

$\phi \rightarrow$ gradient of Potential

It shows that E field is caused by potential gradient.

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From fig 7a), the charge densities are

$$\left.\begin{aligned} \rho(x) &= -qN_a \quad ; \quad -x_p < x < 0 \\ \text{and} \quad \rho(x) &= qN_d \quad ; \quad 0 < x < x_n \end{aligned}\right\} \quad -②$$

. E-field in the p-region is found by integrating eqn①,

$$E = \int \frac{\rho(x)}{\epsilon_{si}}\, dx$$

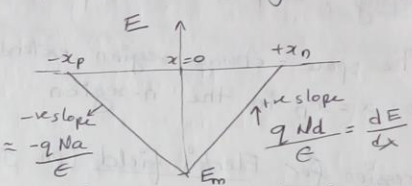

fig: Electric field in the depletion region for a uniformly doped pn junction.

$$E = -\int \frac{qN_a}{\epsilon_{si}}\, dx$$

$$E = -\frac{qN_a}{\epsilon_{si}}\, x + C_1 \quad - ③$$

. The E-field is zero in the neutral p-region for $x < -x_p$, since currents are zero in thermal equilibrium.

∴ Putting $\boxed{E = 0 \; ; \; \text{at } x = -x_p}$

E-field in p-region is

$$E = 0 = -\frac{qN_a x_p}{\epsilon_{si}} + C_1$$

$$C_1 = -\frac{qN_a x_p}{\epsilon_s} \quad - ④$$

$$\therefore \boxed{E = -\frac{qN_a}{\epsilon_{si}}(x + x_p)} \quad \text{for } -x_p \le x \le 0. \quad - ⑤$$

Similarly, the electric field in n-region is given by

$$E = \int \frac{(q N_d)}{\epsilon_{si}} dx = \frac{q N_d x}{\epsilon_{si}} + C_2 \quad -(5)$$

At $x = x_n$, $E = 0$ ($\because$ E-field field can be assumed to be zero in neutral n-region)

$$0 = \frac{q N_d x_n}{\epsilon_{si}} + C_2$$

$$C_2 = -\frac{q N_d x_n}{\epsilon_{si}} \quad -(6)$$

From (5) and (6), we get

$$\boxed{E = -\frac{q N_d}{\epsilon_{si}}(x_n - x)} \quad \text{for } 0 \le x \le x_n \quad -(7)$$

. Maximum electric field occurs at junction (ie $x = 0$), becoz this is where; we have a change from the charge to -ve charge.

$\therefore$ At $x = 0$; we have (From (5) and (7)),

$$\boxed{E_m = \frac{-q N_a}{\epsilon_{si}} \cdot x_p = \frac{-q N_d}{\epsilon_{si}} x_n} \quad -(8)$$

ie $\boxed{N_a x_p = N_d x_n}$ — (9)

Equation (9) states that the number of -ve charges per unit area in the p region is equal to the number of positive charges per unit area in the n-region.

Now, Electric potential = $\int$ Electrical field. $\left(E = \frac{\partial \phi}{\partial x}\right)$ 14

ie $\phi(x) = \int E\, dx$

Important characteristics of E-field:- (Reference)

a) It is directed from n to p region.

b) For uniformly doped pn junction, the E-field is a linear function of distance through the junction

c) It's maximum (magnitude) occurs at the junction (i.e at $x = 0$)

d) An built-in E-field exists in the depletion region, even when no voltage is applied between p and n regions.

. Derivation for Potentials in the junction (Refer fig 7 c)

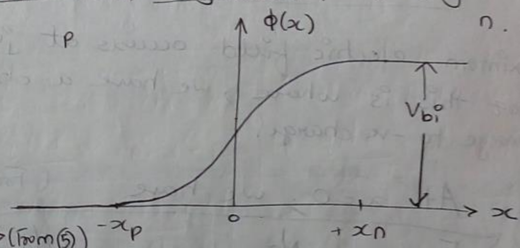

fig: Electric potential through the space charge region of a uniformly doped pn junction.

· In the p-region, the electric potential is

$\phi(x) = -\int E(x)\, dx$

$\phi(x) = \int \frac{q N_a}{\epsilon_s}(x + x_p)\, dx$ → (From (5))

$$\phi(x) = \frac{q N_a}{\epsilon_s}\left[\frac{x^2}{2} + x_p . x\right] + C_3 \quad \text{— (10)}$$

At $x = -x_p$; $\phi(x) = 0$

We are interested in potential difference across the junction, so we assume potential to be zero at ($x = -x_p$.)

$$\therefore \quad 0 = \frac{q N_a}{\epsilon_s}\left[\frac{x_p^2}{2} - x_p^2\right] + C_3$$

$$C_3 = \frac{q N_a}{2\epsilon_s} x_p^2 \quad \text{— (11)}$$

$$\therefore \boxed{\phi_p(x) = \frac{q N_a}{2\epsilon_{si}} (x+x_p)^2} \quad --- -x_p \le x \le 0 \quad - (12)$$

→ Electric potential in p-region.

In n-region, $\phi(x) = -\int E(x)\,dx$

$$\phi(x) = \int \frac{q N_d}{\epsilon_{si}} (x_n - x)\,dx \qquad \text{(From (7))}$$

$$\therefore \phi(x) = \frac{q N_d}{\epsilon_{si}} \left[x_n . x - \frac{x^2}{2}\right] + C_4 \quad - (13)$$

Since potential $\phi(x)$ is a continous function, therefore eq^n (13) must be equal to eq^n (12). at the metallurgical junction or at $x=0$ ⇒

$$C_4 = \frac{q N_a}{2\epsilon_{si}} x_p^2 \quad - (14)$$

∴ Electric potential in n-region is,

$$\phi_n(x) = \frac{q N_d}{\epsilon_{si}} \left[x_n . x - \frac{x^2}{2}\right] + \frac{q N_a}{2\epsilon_{si}} x_p^2 \quad --0 \le x \le x_n \quad - (15)$$

· Magnitude of built-in potential, V_{bi} is the value of $\phi(x)$ at $x = x_n$ & is given by, (from 15).

$$V_{bi} = |\phi_{(x=x_n)}| = \frac{q}{2\epsilon_{si}} \left[N_d x_n^2 + N_a x_p^2\right] \quad - (16)$$

· From fig 7 c), plot of potential variation shows a quadratic dependence on distance.

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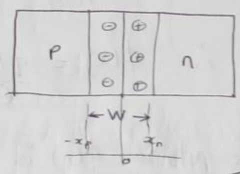
• Derivation for Space-charge width (W)
(for zero applied voltage in pn junction)

Refer Figure (7),

Space-charge width (w) → distance that the space charge region extends into the p and n regions from the junction.

We know, that from (9)



$$N_a x_p = N_d x_n$$

$$\Rightarrow x_p = \frac{N_d x_n}{N_a} \quad -(17)$$

$$x_n = \frac{N_a}{N_d} x_p \rightarrow 17.b$$

∴ Substituting eqⁿ (17) into eqⁿ (16) & solving for x_n we get.

$$V_{bi} = \frac{q}{2\epsilon_s}\left[N_d x_n^2 + \left(\frac{N_d}{N_a}\right)^2 x_n^2\right]$$

$$\text{ie } V_{bi} = \frac{q}{2\epsilon_s}\left(\frac{N_d}{N_a}\right) x_n^2 \left[N_a + N_d\right]$$

$$\Rightarrow x_n = \sqrt{\frac{2\epsilon_s \cdot V_{bi}}{q}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a+N_d}\right]} \quad -(18)$$

Eqⁿ 18 gives width of depletn region, x_n extending into n type region for the case of zero applied voltage.

Similarly, if we solve for x_n from (17)b & substitute into eqⁿ (16), we get

$$x_p = \sqrt{\frac{2\epsilon_s \cdot V_{bi}}{q}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a+N_d}\right]} \quad -(19)$$

where, x_p is the width of depletion region extending into the p region (for zero applied voltage case)

$\therefore$ Total depletion or space charge width (w) is

$$W = x_n + x_p \quad — (20)$$

Using (18) and (19) values in (20), we get.

$$\boxed{W = \sqrt{\frac{2\epsilon_{si} \cdot V_{bi}}{q}\left[\frac{N_a + N_d}{N_a N_d}\right]}} \rightarrow \text{Space-charge width} \quad — (21)$$

where, $\boxed{V_{bi} = \frac{KT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right)}$ — built-in voltage for pn junction

Equation (8) represents the maximum electric field present at pn junction (for zero applied voltage)

ie $\boxed{E_m = \frac{-q N_a}{\epsilon_{si}} . x_p = \frac{-q N_d}{\epsilon_{si}} . x_n}$ — (8)

Using equation (18) in eq^n (8), we get

$$E_m = -\frac{q}{\epsilon_{si}} \sqrt{\frac{2\epsilon_{si} . V_{bi}}{q}\left(\frac{N_a N_d}{N_a + N_d}\right)}$$

$$\boxed{E_m = -\frac{2V_{bi}}{W}} \quad — (22) \; ; \quad \boxed{W = \sqrt{\frac{2\epsilon_{si} . V_{bi}}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}}$$

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We Know, $W = x_p + x_n$ (From 20) 18

Also, From eq^n (9), $N_a x_p = N_d x_n$

$$W = x_n + \frac{N_d}{N_a} x_n = x_n \left[\frac{N_a + N_d}{N_a} \right]$$

So, if we Know W, we can determine x_n and x_p, as

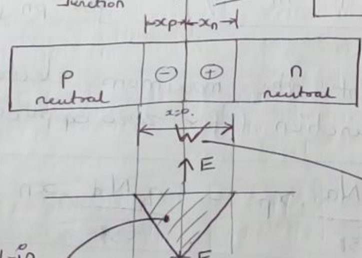
$$\boxed{\begin{aligned} |x_n| &= \left(\frac{N_a}{N_a+N_d}\right) W \\ |x_p| &= \left(\frac{N_d}{N_a+N_d}\right) W \end{aligned}} \quad \text{—} (23)$$

• Summary of PN Junction analysis for Zero-applied voltage.

Junction → $\boxed{N_a x_p = N_d x_n}$

• pn Junction is separated into neutral and space-charge regions.

Two → neutral regions are field-free.



$$W = \sqrt{\frac{2\epsilon_{si} \cdot V_{bi}}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

Area under build-in E-field is Potential.

$$\boxed{V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)}$$

$E_m \rightarrow -\frac{2V_{bi}}{W}$ (we consider only magnitude sometimes)

$\therefore E_m = \left| \frac{2V_{bi}}{W} \right|$

• Space-charge region is assumed to be completed of free carriers.

Peak Electric field occurs at pn Junct^n ($x = 0$) and in space-charge region.

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—x—