Current gain factors:

ie We will derive formulas for γ, b and δ in terms of electrical and geometrical parameters of the transistor.

(The results of these derivations will show how the various parameters in the BJT influence the electrical properties of the device and will point the way to design a "good" BJT.)

A] Expression for γ :-

$$\gamma = \frac{I_{En}}{I_{En} + I_{Ep}} = \frac{1}{\left(1 + \frac{I_{Ep}}{I_{En}}\right)} \quad — ①$$

(Recollect in minority carrier distribution for forward-active mode, I_{En} is in negative x direction).

Now,
$$I_{Ep} = -q D_E A_E \left.\frac{d(\delta p_E(x'))}{dx'}\right|_{x'=0} \quad — ②$$

where,
$$\delta p_E(x') \approx \frac{p_{E0}\left[e^{\frac{V_{BE}}{V_T}} - 1\right] \sinh\left(\frac{W_E - x'}{L_E}\right)}{\sinh\left(\frac{W_E}{L_E}\right)} \quad — ③$$

Now, differentiating eqn ③ w.r.t x', we get

$$\frac{d(\delta p_E(x'))}{dx'}\bigg|_{x'=0} = \frac{-P_{E0}\left[e^{\frac{V_{BE}}{V_T}}-1\right]\cosh\left(\frac{W_E}{L_E}\right)}{L_E\,\sinh\left(\frac{W_E}{L_E}\right)}$$

Now, equation ② becomes,

$$I_{Ep} \approx q\frac{D_E A_E}{L_E} P_{E0}\left[e^{\frac{V_{BE}}{V_T}}-1\right]\frac{1}{\tanh\left(\frac{W_E}{L_E}\right)} \quad — ④$$

Now, $I_{En} \approx -q A_E D_B \dfrac{d(\delta n_B(x))}{dx}\bigg|_{x=0}$ — ⑤

where,

$$\delta n_B(x) \approx \frac{n_{B0}\left\{\left[e^{\frac{V_{BE}}{V_T}}-1\right]\sinh\left(\frac{W_B-x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)\right\}}{\sinh\left(\frac{W_B}{L_B}\right)}$$

$$\therefore \frac{d(\delta n_B(x))}{dx}\bigg|_{x=0} \approx -\frac{n_{B0}}{L_B}\left\{\frac{\left[e^{\frac{V_{BE}}{V_T}}-1\right]\cosh\left(\frac{W_B}{L_B}\right)}{\sinh\left(\frac{W_B}{L_B}\right)} + \frac{1}{\sinh\left(\frac{W_B}{L_B}\right)}\right\}$$

$\therefore$ Equation ⑤ becomes,

$$\left\{\because \cosh\left(\frac{x}{L_B}\right)\bigg|_{x=0} = 1\right\}$$

$$I_{En} \approx \frac{q D_B n_{B0} A_E}{L_B}\left\{\frac{\left(e^{\frac{V_{BE}}{V_T}}-1\right)}{\tanh\left(\frac{W_B}{L_B}\right)} + \frac{1}{\sinh\left(\frac{W_B}{L_B}\right)}\right\} \quad — ⑥$$

If we assume B-E junction is forward bias

ie $V_{BE} \gg V_T$ ie $e^{\left(\frac{V_{BE}}{V_T}\right)} \gg 1$

and also $\left[\dfrac{\left[e^{\left(\frac{V_{BE}}{V_T}\right)} - 1\right]}{\tanh\left(\frac{W_B}{L_B}\right)} \gg \dfrac{1}{\sinh\left(\frac{W_B}{L_B}\right)} \right]$

Equation 6 becomes,

$$I_{En} \simeq q \frac{D_B\, n_{BO} A_E}{L_B} \left[\frac{\left(e^{\frac{V_{BE}}{V_T}} - 1\right)}{\tanh\left(\frac{W_B}{L_B}\right)} \right] \quad — (7)$$

Now, from eqn 1, 4 and 7, we get

$$\therefore \gamma = \frac{1}{1 + \frac{I_{Ep}}{I_{En}}}$$

$$= \frac{1}{1 + \left[\dfrac{\left\{ \left(q \frac{D_E A_E}{L_E} \right) P_{E0} \cancel{\left(e^{V_{BE}/V_T} - 1 \right)} \frac{1}{\tanh\left(\frac{W_E}{L_E}\right)} \right\}}{\left\{ q \frac{D_B n_{BO} A_E}{L_B} \cancel{\left(e^{\frac{V_{BE}}{V_T}} - 1 \right)} \frac{1}{\tanh\left(\frac{W_B}{L_B}\right)} \right\}} \right]}$$

$$\boxed{\gamma = \frac{1}{1 + \frac{P_{E0} D_E L_B}{n_{BO} D_B L_E} \frac{\tanh(W_B/L_B)}{\tanh(W_E/L_E)}}} \quad — (8)$$

If both $\left(\frac{W_B}{L_B}\right) << 1$ & $\left(\frac{W_E}{L_E}\right) << 1$, then we can use approximation $\tanh\left(\frac{W_B}{L_B}\right) \approx \frac{W_B}{L_B}$, $\tanh\left(\frac{W_E}{L_E}\right) \approx \frac{W_E}{L_E}$

∴ Equation (8) becomes,

$$\boxed{\gamma \approx \frac{1}{1 + \frac{N_B D_E W_B}{N_E D_B W_E}}} \quad — (9)$$

∵ $p_{E0} \approx \frac{n_i^2}{N_E}$ and $n_{B0} \approx \frac{n_i^2}{N_B}$

For γ to be close to one, $\boxed{N_E >> N_B}$

(This condition means that many more electrons from Emitter than holes from ptype base will be injected across B-E space-charge region and reach the collector.)

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B] Expression for b (base transport factor) :

$$b = \frac{I_{Cn}}{I_{En}} \quad -①$$

$$I_{Cn} = -q\, D_B A_B \left.\frac{d(\delta n_B(x))}{dx}\right|_{x=W_B} \quad -②$$

From previous derivation eq^n ⑥,

$$I_{Cn} = \frac{q D_B A_B n_{BO}}{L_B}\left[\frac{\left(e^{\frac{V_{BE}}{V_T}}-1\right)}{\sinh\left(\frac{W_B}{L_B}\right)} + \frac{1}{\tanh\left(\frac{W_B}{L_B}\right)}\right] \quad -③$$

Also, I_{En} expression from eq^n ⑦, we get

$$I_{En} \approx \frac{q D_B A_B n_{BO}}{L_B}\left[\frac{\left(e^{\frac{V_{BE}}{V_T}}-1\right)}{\tanh\left(\frac{W_B}{L_B}\right)}\right] \quad -④$$

If we assume, B-E junction is sufficiently biased

ie $V_{BE} >> V_T$ ie $e^{\frac{V_{BE}}{V_T}} >> 1$

ie $$I_{Cn} = \frac{q D_B A_B n_{BO}}{L_B}\left[\frac{\left(e^{\frac{V_{BE}}{V_T}}-1\right)}{\sinh\left(\frac{W_B}{L_B}\right)}\right]$$

Now $b = \frac{I_{Cn}}{I_{En}}$

$$= \frac{\cancel{\frac{q D_B A_B n_{BO}}{L_B}}\left(\frac{\cancel{\left(e^{\frac{V_{BE}}{V_T}}-1\right)}}{\sinh\left(\frac{W_B}{L_B}\right)}\right)}{\cancel{\frac{q D_B A_B n_{BO}}{L_B}}\left(\frac{\cancel{\left(e^{V_{BE}/V_T}-1\right)}}{\tanh(W_B/L_B)}\right)}$$

$$\therefore b \approx \frac{\tanh(W_B/L_B)}{\sinh(W_B/L_B)}$$

$$\boxed{b \approx \frac{1}{\cosh\left(\frac{W_B}{L_B}\right)}} \quad -(5)$$

For $W_B << L_B$, we expand cosh function in Taylor series, so that

$$b = \frac{1}{\cosh\left(\frac{W_B}{L_B}\right)} \approx \frac{1}{1+\frac{1}{2}\left(\frac{W_B}{L_B}\right)^2} \quad \text{(neglecting higher terms)}$$

$$\boxed{b \approx 1-\frac{1}{2}\left(\frac{W_B}{L_B}\right)^2} \quad -(6)$$

Thus, base transport factor (b) will be close to one if $\boxed{W_B << L_B}$.

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c] Expression for δ :- (Recombination Factor)

$$\delta = \frac{I_{En} + I_{Ep}}{I_{En} + I_R + I_{Ep}} \approx \frac{I_{En}}{I_{En} + I_R} \approx \frac{1}{\left(1 + \frac{I_R}{I_{En}}\right)} \quad \text{— (1)}$$

We have assumed in equation (1), that ($I_{Ep} << I_{En}$)

. Recombination current due to recombination in F.B pn junction is

$$I_R = \frac{q W_{BE} n_i}{2 \tau_0} \exp\left(\frac{V_{BE}}{2V_T}\right)$$

$$\boxed{I_R \approx I_{ro} \, e^{\left(\frac{V_{BE}}{2V_T}\right)}} \quad \text{— (2)}$$

where, $W_{BE} \rightarrow$ B-E space charge width.

Also, $I_{En} = I_s \, e^{\left(\frac{V_{BE}}{V_T}\right)}$ — (3)

where, $I_s = \dfrac{q D_B n_{BO}}{L_B \tanh\left(\frac{W_B}{L_B}\right)}$

$$\therefore \quad \delta = \frac{1}{1 + \frac{I_R}{I_{En}}}$$

$$\delta = \frac{1}{1 + \left(\frac{I_{ro} \, e^{(V_{BE}/2V_T)}}{I_s \, e^{(V_{BE}/V_T)}}\right)} \quad \text{— (4)}$$

$$\therefore \boxed{\delta \approx \frac{1}{1+\left[\frac{I_{ro}}{I_S} e^{-\left(\frac{V_{BE}}{2V_T}\right)}\right]}} \quad — (5)$$

Thus, from eq^n (5), δ is a function of B-E voltage V_{BE}.

As V_{BE} ↑ses, the recombination current (I_R) becomes less dominant and δ tends to unity.

Summary:

$$\boxed{\alpha = \frac{I_C}{I_E}}$$

Common-base current gain

$$\boxed{\beta = \frac{I_C}{I_B}}$$

$$\boxed{\beta = \frac{\alpha}{1-\alpha}}$$

Common-emitter current gain.

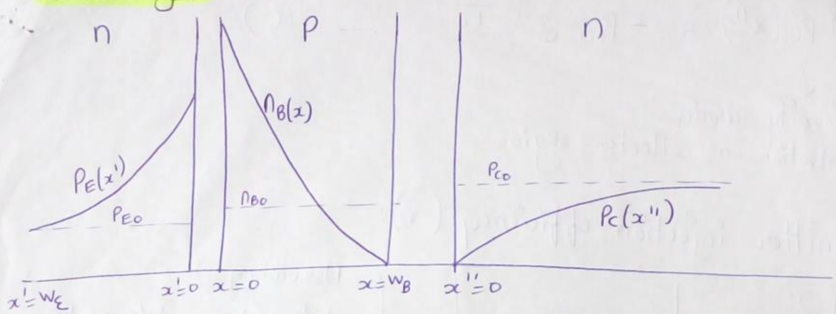
$$I_E = I_B + I_C$$

$$\Rightarrow \frac{I_E}{I_C} = \frac{I_B}{I_C} + 1$$

$$\text{i.e.} \ \frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\boxed{\alpha = \frac{\beta}{1+\beta}}$$

Summary:



$$\delta n_B(x) = \frac{n_{BO}\left\{\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\sinh\left(\frac{W_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)\right\}}{\sinh\left(\frac{W_B}{L_B}\right)} \quad \text{— (A)}$$

Excess min carrier concentration in the Base region

If $W_B \ll L_B$, $\sinh\left(\frac{W_B}{L_B}\right) \approx \frac{W_B}{L_B}$

$$\delta n_B(x) \approx \frac{n_{BO}}{W_B}\left[\left(e^{\frac{V_{BE}}{V_T}} - 1\right)(W_B - x) - x\right] \quad \text{— (A1)}$$

$$\delta P_E(x') \approx \frac{P_{EO}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\sinh\left(\frac{W_E - x'}{L_E}\right)}{\sinh\left(\frac{W_E}{L_E}\right)} \quad \text{— (B)}$$

If W_E is small (ie for a thin emitter)

$$\delta P_E(x') \approx \frac{P_{EO}}{W_E}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)(W_E - x') \quad \text{— (B1)}$$

Excess min. carrier concentration in Emitter region

$$\delta p_c(x'') \approx -p_{co}\, e^{\left(-\frac{x''}{L_c}\right)} \quad \text{— (C)}$$

Excess Minority carrier concentration in collector region

· Emitter injection efficiency (γ):

$$\boxed{\gamma \approx \frac{1}{1 + \frac{N_B D_E W_B}{N_E D_B W_E}}}$$

(Assuming: $W_B \ll L_B$, $W_E \ll L_E$)

· Base transport factor (b):

$$\boxed{b \approx \frac{1}{1 + \frac{1}{2}\left(\frac{W_B}{L_B}\right)^2}}$$

($W_B \ll L_B$)

· Recombination factor (δ):

$$\boxed{\delta \approx \frac{1}{1 + \left(\frac{I_{ro}}{I_s} \exp\left(\frac{-V_{BE}}{2V_T}\right)\right)}}$$

· $\boxed{\alpha = \gamma b \delta}$, $\boxed{\beta = \frac{\alpha}{1-\alpha}}$

—x—