Module 02: Bipolar Junction Transistor

01 R
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Topic: 1] Equivalent Circuit Models for BJT
2] Non-Ideal Effects

· In order to analze or design a transistor circuit, one needs a mathematical model or equivalent circuit of the transistor.

Models for BJT
- A] Eber's moll model
- B] Gummel-Poon model
- C] Hybrid-π model

→ These 3 equivalent circuit models can replace transistors in a circuit.

* Eber's moll model can be used for a transistor biased in any of its operating modes. & are particularly used in switching applications.

* Gummel-Poon model also take into account non-ideal effects in a BJT, which are ignored in Eber's moll model.
(Early effects & high level injection)

* Hybrid-pi model is applied when the transistors are operated in a small signal linear amplifier & takes into account frequency effests within the tranistor.

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* Eber's - Moll model:

- A BJT consists of two interacting junction diodes and Eber's - moll model uses this interaction as its basic fundamental foundation and is applicable in any of BJT operating modes

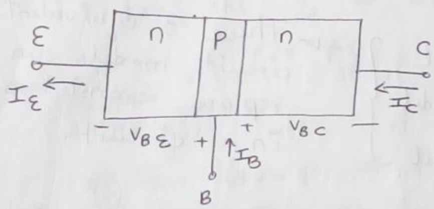

fig a. Current direction's and voltage polarity definitions for Ebers-moll model.

From fig a, $I_E = I_B + I_C$ — ①

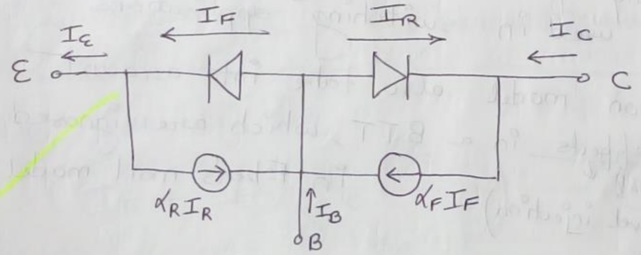

fig b: Eber's moll model for npn BJT

extra!

{ Idea here is to evaluate terminal currents ie I_E, I_B, I_C, if voltage applied between the two junction's are known ie V_{BE} and V_{BC} and vice-versa. That the idea behind this model! }

From fig (b), we can write

A) For $V_{BE} \neq 0$, $V_{BC} \neq 0$, cases.

$$\left.\begin{aligned} I_C &= \alpha_F I_F - I_R \\ I_E &= I_F - \alpha_R I_R \end{aligned}\right] \begin{aligned} &— (2) \\ &— (3) \end{aligned}$$

Extra!

B) For $V_{BE} = 0$, $\Rightarrow I_F = 0$

$$\beta_R = \frac{\alpha_R}{1-\alpha_R}$$

Reverse current gain

C) For $V_{BC} = 0 \Rightarrow I_R = 0$

then $I_C = \alpha_F I_F$, $I_E = I_F$

$$\boxed{\begin{aligned} I_E &= I_C + I_B \\ I_B &= I_E - I_C \end{aligned}}$$

$$I_B = (1-\alpha_F) I_F$$

$$\therefore \frac{I_C}{I_B} = \frac{\alpha_F}{1-\alpha_F} = \beta_F \rightarrow \text{forward current gain}$$

Also, from fig (b),

$$I_F = I_{ES}\left(e^{\frac{V_{BE}}{V_T}} - 1\right) \quad — (4)$$

$$I_R = I_{CS}\left(e^{\frac{V_{BE}}{V_T}} - 1\right) \quad — (5)$$

where, I_{ES} is reverse-bias B-E junction current

I_{CS} is reverse saturation current for B-C junction.

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Substitutes eqⁿ 4 and 5 values in to eqⁿ 2, we get

$$\boxed{I_C = \alpha_F I_{ES}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right)} \quad — (6)$$

Substitute eqⁿ 4 & 5 values into eqⁿ 3, we get

$$\boxed{I_E = I_{ES}\left(e^{V_{BE}/V_T}-1\right) - \alpha_R I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right)} \quad — (7)$$

Equation 6 and 7 represent the Ebers-Moll Equations, which are used to develop Ebers-moll equivalent circuit.

- The equivalent circuit consists of two current sources and two ideal pn junctions as shown in fig (b). Each current source is dependent upon the voltages across other junction.
- The Ebers-moll model has 4 parameters:

  α_R, α_R, I_{ES} and I_{CS}.

- However, these 4 parameters are not independent (only 3 are independent) quantities but are inter-related through the relation.

$$\boxed{\alpha_F I_{ES} = \alpha_R I_{CS}} \quad — (8)$$

Reciprocity relationship

∵ Ebers-moll model is valid for any mode 05 of operation of BJT. For example, in the saturation mode, both B-E and B-C junctions are Forward-bias, ie V_{BE} & V_{BC} used in equations (6) and (7) are positive (ie $V_{BC} > 0$, $V_{BE} > 0$)

Extra!

- The B-E voltage will be a known parameter, since we will apply a voltage across this junction
- The F.B, B-C voltage is a result of driving the transistor into saturation and is the unknown to be determined from the Ebers-moll equations

- In a lot of applications involving BJT, C-E voltage at saturation ie $V_{CE(sat)}$ is an important quantity (of interest).

$$V_{CE(sat)} = V_{BE} - V_{BC} \quad \text{—— (9)}$$

We will find an expression for $V_{CE(sat)}$: by combining Eber's-moll equations.

Solving for V_{BE} :

Rewritting eq^n (6) and putting value of I_{CS} from eq^n eq^n

ie $\boxed{I_{CS} = \frac{\alpha_F I_{ES}}{\alpha_R}}$

ie $$I_C = \alpha_F I_{ES}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \frac{\alpha_F I_{ES}}{\alpha_R}\left(e^{\frac{V_{BC}}{V_T}}-1\right) \quad \text{--- (10)}$$

ie $$\alpha_R I_C = \alpha_R \alpha_F I_{ES}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \alpha_F I_{ES}\left(e^{\frac{V_{BC}}{V_T}}-1\right) \quad \text{--- (11)}$$

Now from eq^n (1) and (7), we get $\{\boxed{I_E = I_C + I_B}$

$$I_{ES}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \alpha_R I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right) = I_B + I_C$$

ie using relation from eq^n (8), we get

$$I_{ES}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \boxed{\alpha_F I_{ES}}\left(e^{\frac{V_{BC}}{V_T}}-1\right) = I_B + I_C \quad \text{--- (12)}$$

Eq^n (11) can be written as,

$$\alpha_F I_{ES}\left(e^{\frac{V_{BC}}{V_T}}-1\right) = \alpha_R \alpha_F I_{ES}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \alpha_R I_C \quad \text{--- (13)}$$

Put eq^n (13) in eq^n (12), we get (ie eliminating V_{BC} term)

$$I_{ES}\left(e^{V_{BE}/V_T}-1\right) - \left[\alpha_R \alpha_F I_{ES}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \alpha_R I_C\right] = I_B + I_C$$

ie $$I_{ES}\left(e^{V_{BE}/V_T}-1\right)\left(1-\alpha_R \alpha_F\right) = I_B + I_C\left(1-\alpha_R\right)$$

ie $$\left(e^{V_{BE}/V_T}-1\right) = \frac{I_B + I_C\left(1-\alpha_R\right)}{I_{ES}\left(1-\alpha_R \alpha_F\right)}$$

$$e^{\frac{V_{BE}}{V_T}} = \frac{I_B + I_C(1-\alpha_R)}{I_{ES}(1-\alpha_R\alpha_F)} + 1$$

$$\therefore \boxed{V_{BE} = V_T \ln\left[\frac{I_B + I_C(1-\alpha_R) + I_{ES}(1-\alpha_R\alpha_F)}{I_{ES}(1-\alpha_R\alpha_F)}\right]} \quad \text{— (A)}$$

Similarly, Solving for V_{BC}:

From eq^n (6) and (8), we get $\boxed{\alpha_F I_{ES} = \alpha_R I_{CS}}$

$$I_C = \alpha_R I_{CS}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right) \quad \text{— (14)}$$

Using eq^n (7) & substituting $\boxed{I_{ES} = \frac{\alpha_R I_{CS}}{\alpha_F}}$ we get

$$\therefore I_E = \frac{\alpha_R I_{CS}}{\alpha_F}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \alpha_R I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right)$$

ie $\alpha_F I_E = \alpha_R I_{CS}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \alpha_F\alpha_R I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right)$

Now, $I_E = I_C + I_B$,

ie $\alpha_F I_C + \alpha_F I_B = \alpha_R I_{CS}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \alpha_F\alpha_R I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right)$ — (15)

Rewriting eq^n (14), (To Eliminate V_{BE} terms)

$$\alpha_R I_{CS}\left(e^{\frac{V_{BE}}{V_T}}-1\right) = I_C + I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right) \quad \text{— (16)}$$

Substituting eq^n (16) into eq^n (15), we get

$$\alpha_F I_C + \alpha_F I_B = I_C + I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right) - \alpha_R \alpha_F I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right)$$

ie $$I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right)\left[1-\alpha_R\alpha_F\right] + I_C(1-\alpha_F) - \alpha_F I_B = 0$$

ie $$I_{CS}(1-\alpha_R\alpha_F)\left(e^{\frac{V_{BC}}{V_T}}-1\right) = \alpha_F I_B - I_C(1-\alpha_F)$$

ie $$\left(e^{\frac{V_{BC}}{V_T}}-1\right) = \frac{\alpha_F I_B - I_C(1-\alpha_F)}{I_{CS}(1-\alpha_R\alpha_F)}$$

$$\therefore e^{\frac{V_{BC}}{V_T}} = \frac{\alpha_F I_B - I_C(1-\alpha_F)}{I_{CS}(1-\alpha_R\alpha_F)} + 1$$

$$\therefore \boxed{V_{BC} = V_T \ln\left[\frac{\alpha_F I_B - I_C(1-\alpha_F) + I_{CS}(1-\alpha_F\alpha_R)}{I_{CS}(1-\alpha_F\alpha_R)}\right]} \quad \text{— (B)}$$

$$V_{CE(sat)} = V_{BE} - V_{BC}$$ (From eq^n (A) & (B), we get)

ie $$V_{CE(sat)} = V_t \ln\left[\frac{I_C(1-\alpha_R) + I_B}{\alpha_F I_B - (1-\alpha_F) I_C} \times \frac{I_{CS}}{I_{ES}}\right]$$

$$\boxed{V_{CE(sat)} = V_t \ln\left[\frac{I_C(1-\alpha_R) + I_B}{\alpha_F I_B - (1-\alpha_F) I_C} \cdot \frac{\alpha_F}{\alpha_R}\right]} \quad \text{— C}$$

($\because \alpha_F I_{ES} = \alpha_R I_{CS}$ From eq^n (8).)

1. A BJT has following significant parameters at T = 300K, 09R

$\alpha_F = 0.98$, $\alpha_R = 0.18$, $I_C = 2mA$; $I_B = 60\mu A$

Calculate collector to emitter voltage at saturation.

Solution:

$$V_{CE(sat)} = V_T \ln\left[\frac{I_C(1-\alpha_R)+I_B}{\alpha_F I_B - (1-\alpha_F)I_C}\left(\frac{\alpha_F}{\alpha_R}\right)\right]$$

$$\text{ie } V_{CE(sat)} = 0.026 \ln\left[\frac{2\times10^{-3}(1-0.18)+60\times10^{-6}}{0.98\times60\times10^{-6}-(1-0.98)\,2\times10^{-3}}\left(\frac{0.98}{0.18}\right)\right]$$

$\boxed{V_{CE(sat)} \simeq 0.16\,V}$ → Ans

Note: Because of log function, $V_{CE(sat)}$ is not a strong function of I_C or I_B.

—X—

* NON-IDEAL EFFECTS IN BJT: 25/08/14 11R

So far we have considered a BJT with

a) uniform doped regions
b) low injections level
c) Constant emitter and base widths
d) Constant energy bandgap
e) Uniform current densities and
f) Junctions which are not in breakdown.

} Ideal conditions for a BJT.

· If any of these ideal conditions are not present, then the transistor properties will deviate from the ideal characteristics derived.

Various non-ideal effects that needs to be investigated there for BJT are including,

a) Base width modulation (Early effect)

b) High level injection

c) Non-uniform Base doping

d) Breakdown voltage

e) Emitter Bandgap narrowing.

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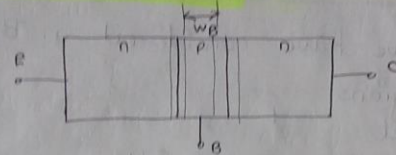
A] **Base width modulation:**



npn transistor

- Uptill now we have assumed that base width 'W_B' was constant.
- However, 'W_B' is a function of B-C voltage V_{BC}, width of depletion region extending into the base region varies with V_{BC}.
- As R.B. V_{BC} ↑ses, B-C depletion region width ↑ses, which reduces W_B.

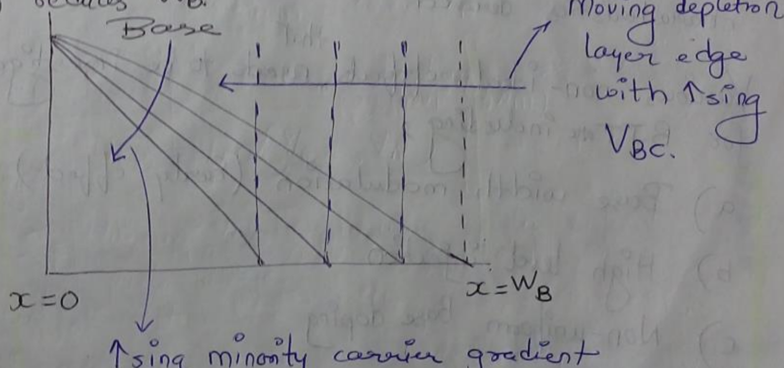


fig 1: Change in Base-width and the change in minority carrier gradient as the B C depletion region width changes.

∴ A change in W_B will change collector current 13
I_C as can be observed in fig1.

. That is, a reduction in base-width (W_B) will cause the gradient in minority carrier to ↑se which in turn causes an ↑se in diffusion current.

. This effect is known as 'Base-width modulation' or 'Early effect'.

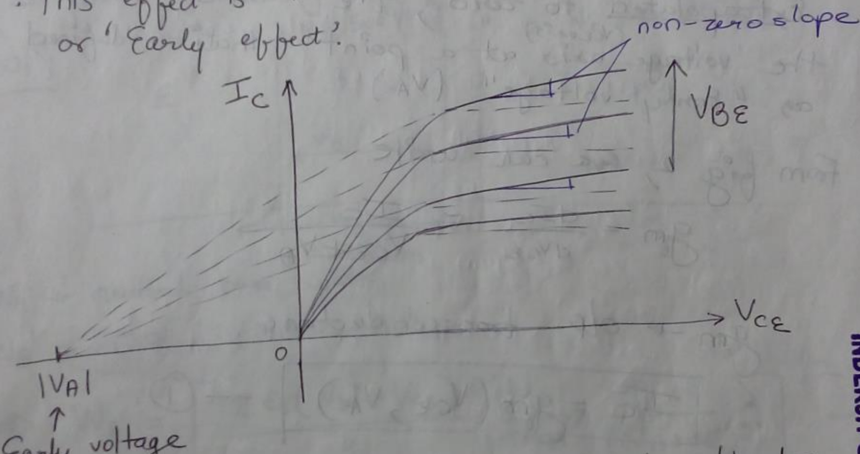


fig 2: I_C versus V_{CE} showing early effect and early voltage.

. In most cases, V_{BE} constant is equivalent to constant base current. Ideally, I_C is independent of V_{BC} thus, slope of curve would be zero & hence o/p Conductance of BJT would be zero.

$\left(\text{ie } g_o = \frac{\Delta I_C}{\Delta V_{CE}} \approx 0\right) \rightarrow$ since $\Delta I_C \Rightarrow$ constant (without BWM)

- However, "BWM" or "Early effect", produces a 'non-zero slope' & give rise to a finite o/p conductance ($g = \frac{\Delta I_C}{\Delta V_{CE}}$)
- From fig 2, if "I_C characteristics" are Extrapolated to zero (VCE axis), the curves intersects the voltage axis at a point that is defined as "Early voltage" (V_A)

From fig 2, we can write

$$g_o = \frac{dI_C}{dV_{CE}} \approx \frac{I_C}{V_{CE}+V_A}$$

$g_o \rightarrow$ o/p sconductance.

$$\therefore \boxed{I_C = g_o (V_{CE}, V_A)} \quad — (1).$$

Eqn (1) shows that collector current is now a function of C-E voltage or the CB voltage.

ie $\boxed{V_{CE} = V_{BE} - V_{BC}}$

— x —

Summary:

As V_{BC} ↑ses, W_B ↓ses
↓W_B causes gradient in minority - carrier concentration to ↑se, which causes an ↑se in diffusion current.
$\therefore$ β also ↑ses.
$\therefore$ I_C ↑ses with ↑sing V_{CE}
⇒ BWM