

\* **Hybrid- $\pi$  Model :**

- Eber's-moll model and Gummel-Poon model do not take into account the junction capacitances associated with BJT.
  - These models therefore, tend to fail when analysis of BJT is done at high frequencies.
  - We consider the various terminals of BJT individually for constructing the model.
- Consider npn BJT in CE configuration.

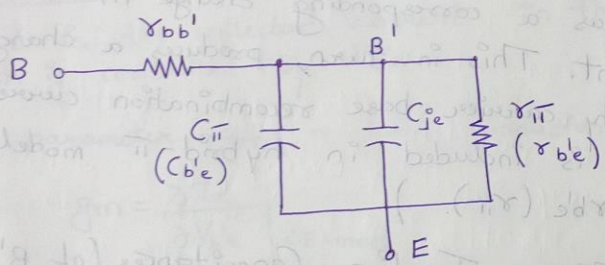


fig a) Components of hybrid- $\pi$  model between base(B) and emitter(E)

• In figure(a), B represents the external base terminal and B' is the active internal part of the base region.

• Resistance  $r_{bb'}$  : It is the series resistance in base between external base terminal B and internal base region B'.

2. Capacitance  $C_{\pi}$  ( $C_{b'e}$ ):

B'-E junction is forward-bias, So  $C_{\pi}$  is the junction diffusion capacitance.

(The excess minority carrier storage in the base region results in the diffusion capacitance  $C_{\pi}$  between B' and E).

3. Resistance  $r_{\pi}$  ( $r_{b'e}$ ): It is the junction diffusion resistance.

(Any small change in the B-E forward bias voltage produces a corresponding change in the emitter current. This in turn, produces a change in the minority carrier base recombination current. This effect is included in hybrid- $\pi$  model as a resistor  $r_{b'e}$  ( $r_{\pi}$ ).

4. Capacitance  $C_{\mu}$  : Junction Capacitance (of B'E junction)

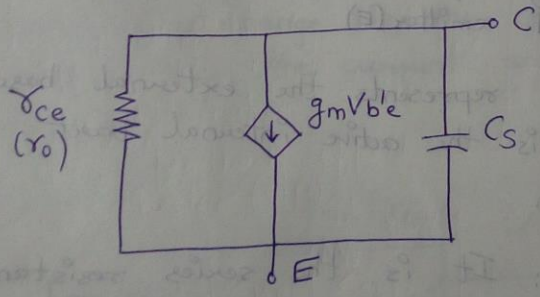


fig (b) Components of hybrid- $\pi$  model between collector (C) and emitter (E)

5. Resistance  $r_{ce}(r_o)$ : The resistance existing between the C and E is represented by  $r_{ce}(r_o)$  ( $r_o$  is the o/p resistance which is the inverse of o/p conductance and is primarily due to Early effect).

6. Capacitance  $C_s$ : It is the junction capacitance of reverse-bias collector substrate junction.

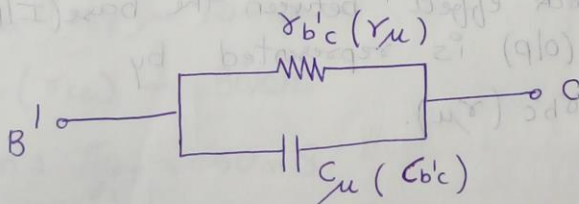
7.  $g_m V_{b'e}$  (Dependent current source): It is the collector current in the transistor, which is controlled by  $V_{b'e}$ .

(The parameter  $g_m$  (mutual conductance)  $\Rightarrow$  is

$$g_m = \left. \frac{\partial I_c}{\partial V_{b'e}} \right|_{CE\text{-mode}}$$

Any small change  $V_{b'e}$  in B-E junction produces a corresponding change in the collector current. This results in the current source indicated by  $g_m V_{b'e}$ .

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fig(c) Components of hybrid- $\pi$  model between Base (B) and collector (C)



8. Capacitance  $C_{\mu}(C_{bc})$ :

Since B-C junction is reverse-biased,  $C_{\mu}(C_{bc})$  is the reverse-bias junction capacitance.

9. Resistance  $r_{\mu}(r_{bc})$ :

It is the reverse-bias diffusion resistance.

Extra: !!

Any change in B-C reverse-bias voltage produces a corresponding change in depletion width of B-C junction.

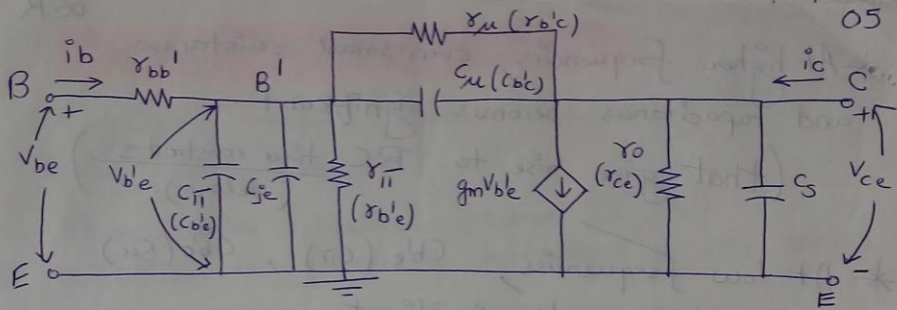
This results in a change in effective base width.

The phenomenon that results in a dependence of effective base width on collector voltage is called "Early effect" or (BWM)  $\rightarrow$  Base-width modulation.

Any change in the effective base width produces a corresponding change in the emitter and collector currents.

This is because a change in the base width results in a change in the slope of minority carrier distribution in the base.

$\rightarrow$  This feedback effect between the base (I/P) and collector (O/P) is represented by resistance's  $r_{bc}(r_{\mu})$ .

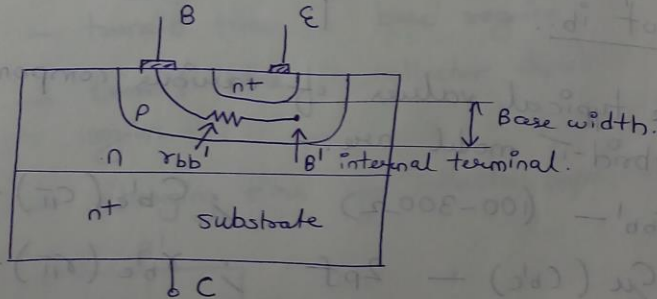


fig(d) Complete Hybrid- $\pi$  model for npn BJT in CE configuration

\* (Valid for both low frequencies and high frequencies)

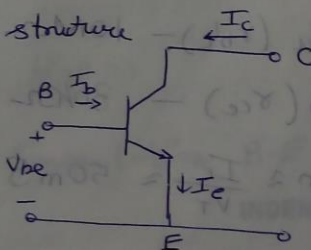
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Extra !!



fig(5) npn BJT device structure

In the model, we have B-E Junction capacitance and B-C junction capacitance shown as  $[C_{b'e}$  and  $C_{b'c}]$

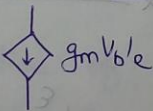


\* At higher frequencies, even small resistances and capacitances become significant.  
(that give rise to RC time constants (delays))

\* At low frequencies,  $C_{b'e}$  ( $C_{\pi}$ ),  $C_{b'c}$  ( $C_{\mu}$ ) and  $r_{bb'}$  are not significant.

Note: In practice, transistor action transfers only the current in  $\beta_{II}$  to the o/p, it does not transfer current through  $C_{\pi}$  to o/p.

∴ Current source is controlled by  $V_{b'e}$  only & not  $i_b$ .



Some typical values of various components in hybrid- $\pi$  model are:-

- $r_{bb'}$  - (100-300 $\Omega$ ) ;  $C_{b'e}$  ( $C_{\pi}$ )  $\sim$  20pf
- $C_{\mu}$  ( $C_{b'c}$ ) - 2pf ;  $r_{b'e}$  ( $r_{\pi}$ ) - 1.5k $\Omega$ .
- $r_{\mu}$  ( $r_{b'c}$ ) - 10M $\Omega$
- $r_o$  ( $r_{ce}$ ) - 30k $\Omega$
- $g_m \approx \frac{I_c}{V_T} \approx 50\text{mS}$



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## \* Frequency Limitations in BJT:

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- Hybrid- $\pi$  model introduces frequency effects through R-C elements.
- Thus, various physical factors in BJT contribute to frequency limitations of the device.

→ Various time-Delay factors in BJT are:-

Total delay time  $\tau_0 (\tau_{ec})$  between emitter and collector is given as:

$$\tau_0 = \tau_e + \tau_b + \tau_d + \tau_c \quad \text{--- (1)}$$

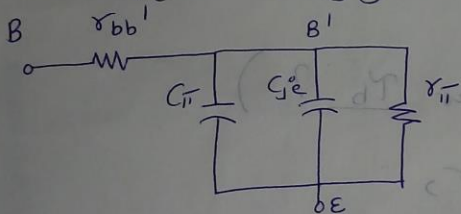
where,

$\tau_e$  - charging time of B-E junction capacitance

$\tau_b$  - transit time in base region.

$\tau_d$  - transit time in collector depletion region

$\tau_c$  - charging time of collector capacitance.



fig(1.a)

The equivalent circuit of forward-bias B-E junction is shown in fig(1.a).

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From the diagram (1.a), capacitance  $C_{je}$  is the junction capacitance.

1] Charging time  $\tau_e$  of B-E junction capacitance is

$$\tau_e = r_e' (C_{je} + C_p) \quad \text{--- (2)}$$

where,  $r_e'$  - diffusion resistance of emitter junction

$C_{je}$  - Emitter junction capacitance.

$C_p$  - represents all the parasitic capacitance between the Base and emitter.

2] The transit time ( $\tau_b$ ) in base region is given by,

$$\tau_b = \frac{W_B^2}{2D_n} \quad \text{--- (3)}$$

where,  $W_B$  - width of the base region.

\* Base transit time ( $\tau_b$ ) is the time required for minority carriers to diffuse across the neutral base region.

→ Extra! (Expression for  $\tau_b$ ?)

• For an npn transistor,

$$J_n = -q n_B(x) v(x)$$

↓  
Electron

current density in base.



where,  $v(x)$  - average velocity of carriers

$$v(x) = \frac{dx}{dt} \Rightarrow dt = \frac{dx}{v(x)}$$

Transit time can be found by integrating,

$$\tau_b = \int_0^{W_B} dt = \int_0^{W_B} \frac{dx}{v(x)} = \int_0^{W_B} \frac{q n_B(x) dx}{(-J_n)} \quad \text{--- (i)}$$

Now,

$$n_B(x) \approx n_{B0} \left[ \exp\left(\frac{V_{BE}}{V_T}\right) \left(1 - \frac{x}{W_B}\right) \right] \quad \text{--- (ii)}$$

$$\text{Now, } J_n = q D_n \frac{dn_B(x)}{dx} \quad \text{--- Diffusion current.} \quad \text{--- (iii)}$$

Solving (i), (ii) and (iii), we can get

$$\tau_b = \frac{W_B^2}{2D_n}$$

Note  $x_B = W_B$

In textbooks,  
they have used  
Base width symbol  
as  $x_B$

3] Collector depletion region transit time ( $\tau_d$ ) is

$$\tau_d = \frac{W_{dc} \text{ (or } x_{dc})}{v_s} \quad \text{--- (4)}$$

Assuming that the electrons in npn device travel across the B-C space charge region with their saturation velocity ( $v_s$ ).

$W_{dc}$  or ( $x_{dc}$ ) is the B-C space-charge width.

4] Charging time of collector capacitance ( $\tau_c$ ) is

$$\tau_c = r_c (C_u + C_s) \quad \text{--- (5)}$$

where,  $C_u$  - B-C junction capacitance.

$r_c$  - Collector region series resistance.

$C_s$  - Collector to substrate capacitance.

\* B-C is reverse-bias so that diffusion resistance in parallel with junction capacitance is very large.

Thus,  $\tau_c$  is a function of  $r_c$

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How frequency affects the transistor performance!

- BJT is a transit-time device. So any changes in  $V_{BE}$  leads to a corresponding change in carrier injected from Emitter into Base region.
- If  $V_{BE} \uparrow$ , additional carriers from emitter needs to be injected into the base region.
- These excess carriers then diffuse through base-region, suffers some recombination's there and finally gets collected in the collector region.

→ Frequency Limitation effect:-

- \* As the frequency of input signal applied  $\uparrow$ , the transit time of the carriers from E to C becomes comparable to the time period of  $\Delta P$  signal.
- \* When this happens, the O/P is no more in phase with the  $\Delta P$  signal.
- \* This leads to a corresponding  $\downarrow$  in the magnitude of current gain.