* Transistor Cut-off frequency: (f_T)

. The current gain as a function of frequency is

$$\alpha = \frac{\alpha_0}{1 + j\left(\frac{f}{f_\alpha}\right)} \quad — ①$$

where, α_0 — Low-freq^n CB current gain
↳ (common base)

f_α — Alpha cut-off frequency.

Now, $f_\alpha = \frac{1}{2\pi \; \tau_D \text{ or } (\tau_{ec})}$ — ②

. When the frequency is equal to the alpha cut-off frequency, the magnitude of CB current gain is $\left(\frac{1}{\sqrt{2}}\right)$ of its low-frequency value.

. We Know, $\beta = \frac{\alpha}{1-\alpha}$.

From ①, when $f \approx f_\alpha$, then

$$|\beta| = \left|\frac{\alpha}{1-\alpha}\right| = \frac{f_\alpha}{f} \quad — ③$$

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From (3) when signal frequency (f) is equal to f_α, the magnitude of C.E gain is unity ie $|\beta| \simeq 1$.

$$\therefore \boxed{f_T = \frac{1}{2\pi \, \tau_D \text{ or } \tau_{cc}}} \quad -(4)$$

Transistor cut-off frequency.

Also, we can write C.E current gain as,

$$\beta = \frac{\beta_0}{1 + j(f/f_\beta)} \quad -(5)$$

where f_β - beta cut-off frequency

It is the frequency at which magnitude of β drops to $\left(\frac{1}{\sqrt{2}}\right)$ of its low-frequency value.

Now, $\beta = \frac{\alpha}{1-\alpha}$

$$\beta = \frac{\dfrac{\alpha_0}{1 + j(f/f_T)}}{1 - \dfrac{\alpha_0}{1+j(f/f_T)}} \approx \frac{\alpha_0}{1 - \alpha_0 + j(f/f_T)}$$

ie $$\beta \approx \frac{\alpha_0}{(1-\alpha_0)\left[1 + j\frac{f}{(1-\alpha_0) f_T}\right]}$$

$$\beta \approx \frac{\beta_0}{1 + j\,\beta_0\left(\frac{f}{f_T}\right)} \quad -(6)$$ where, $\beta_0 = \frac{\alpha_0}{1-\alpha_0} \approx \frac{1}{1-\alpha_0}$.

Comparing (5) and (6), we get

$$\boxed{f_\beta \approx \frac{f_T}{\beta_0}} \quad -(7)$$

Equation (7) relates beta-cut-off frequency to transistor cut-off frequency.

Ex: Calculate the emitter-collector transit time, cut-off frequency of a BJT for a silicon npn transistor at $T = 300K$ with $\beta = 100$, $I_E = 50\mu A$, $C_{je} = 0.4pf$, $C_\mu = 0.05pf$, $X_B = 0.5\mu m$, $D_n = 25 cm^2/s$, $X_{dc} = 2.4\mu m$, $r_c = 20\Omega$, $C_s = 0.1pf$.

⇒ Neglecting parasitic capacitances,

$$\tau_e = r_e' C_{je}$$

where, $r_e' = \frac{V_T}{I_E} = \frac{0.0259}{50 \times 10^{-6}} = 518\,\Omega$

$$\Rightarrow \tau_e = r_e' C_{je}$$
$$\simeq (518) \times 0.4\,pf \simeq 207.2\ psec$$

· Base transit time is,

$$\tau_b = \frac{X_B^2}{2D_n} = \frac{(0.5 \times 10^{-4})^2}{2 \times 25} = 50\ psec$$

· $\tau_d = \frac{W_{dc} \text{ or } (X_{dc})}{v_s}$ Assuming $v_s = 10^7$ cm/s.

$$= \frac{2.4 \times 10^{-4}}{10^7} = 24\ ps$$

· $\tau_c = r_c(C_\mu + C_s)$

$$= 20(0.05 + 0.1)\,pf = 3\ pSec.$$

∴ Emitter-collector transit time

$$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$$
$$= 284.2\ psec.$$

Cut-off frequency of a BJT is

$$f_T = \frac{1}{2\pi \tau_{ec}}$$

$$= \frac{1}{2\pi(284.2 \times 10^{-12})}$$

$$f_T \simeq 0.56 \text{ GHz}$$

Now, beta cut-off frequency (f_β) is

$$f_\beta \approx \frac{f_T}{\beta}$$

$$\approx \frac{0.56 \times 10^9}{100}$$

$$f_\beta \simeq 5.6 \text{ MHz.}$$

—x—