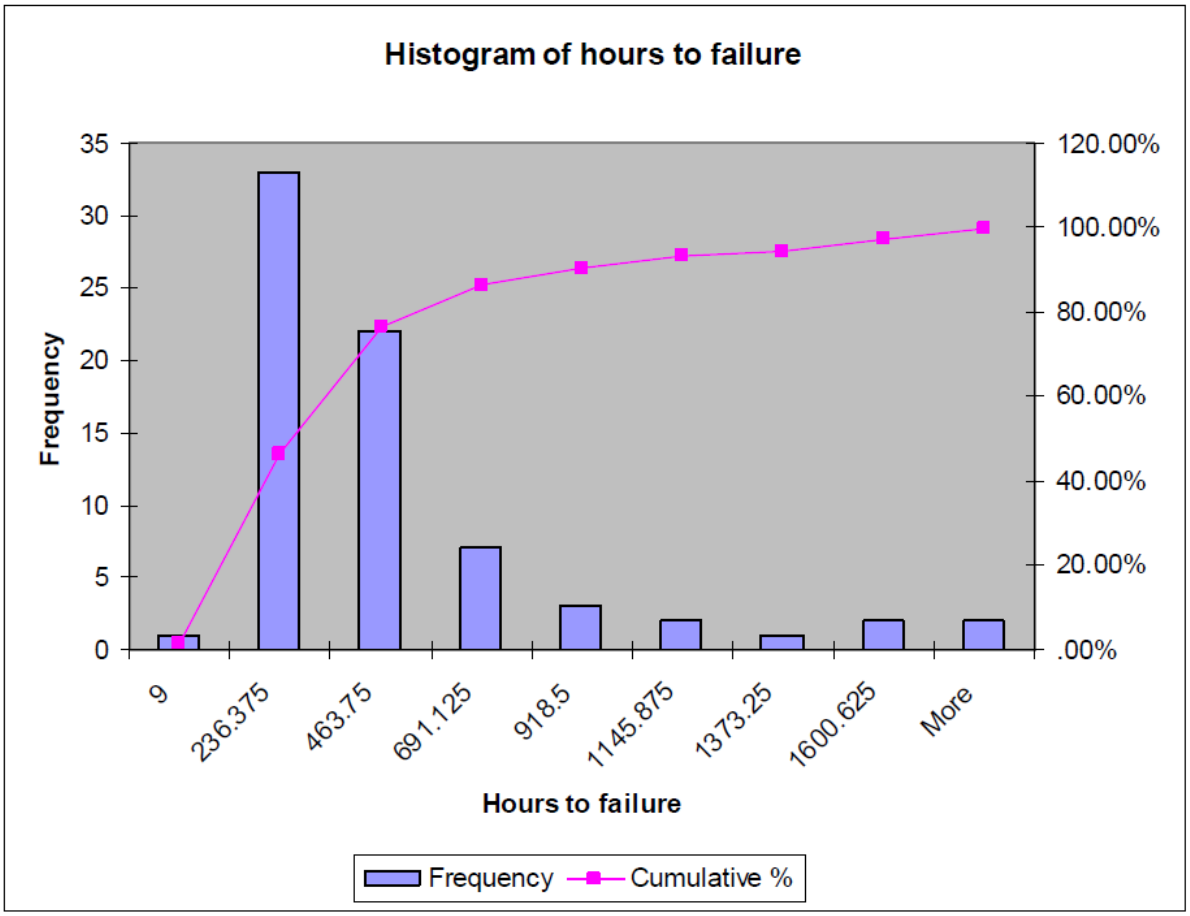
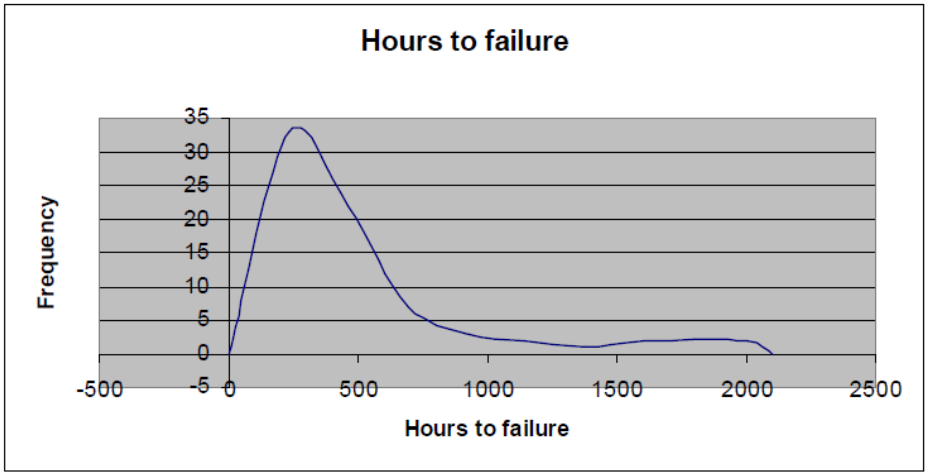
- Sample data from a population of items
- For example:
  - 100 ipods put on test, 12 fail, analyse the times to failure
  - 1000 aircraft engine controllers operating in-service, collect all the times to failure data and analyse
- Not only times but distance or cycles etc.



Probability distribution



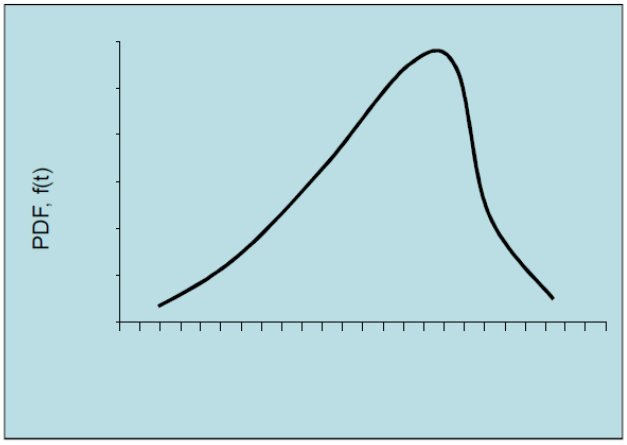
- The area under the curve is equal to 1
- The area under the curve between two values is the probability

Failure Time distributions

- PDF (Probability density function)
- The CDF (Cumulative Distribution Function)
  - The CDF gives the probability that a unit will fail before time t or alternatively the proportion of units in the population that will fail before time t.
- The Survival Function (sometimes known as reliability function)
  - Complement of the CDF.
- The Hazard Function
  - Conditional probability of failing in the next small interval given survival up to time t.

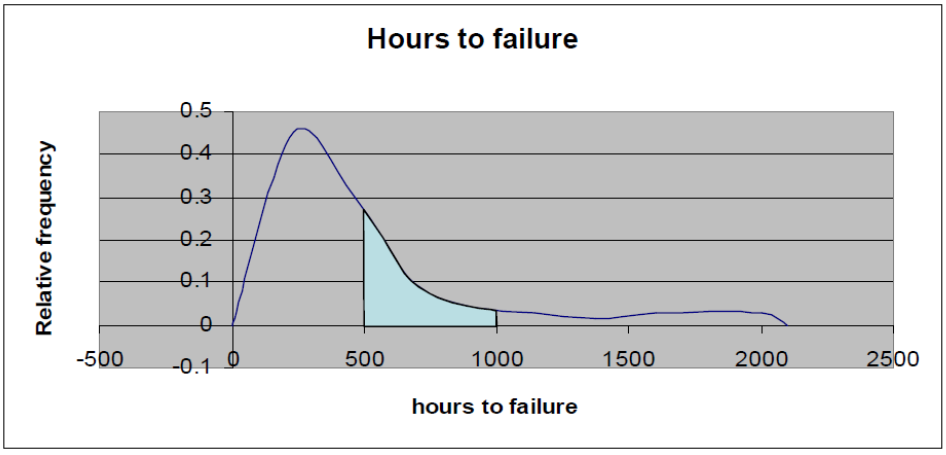
Probability density Function:

- PDF - Probability of falling between two values



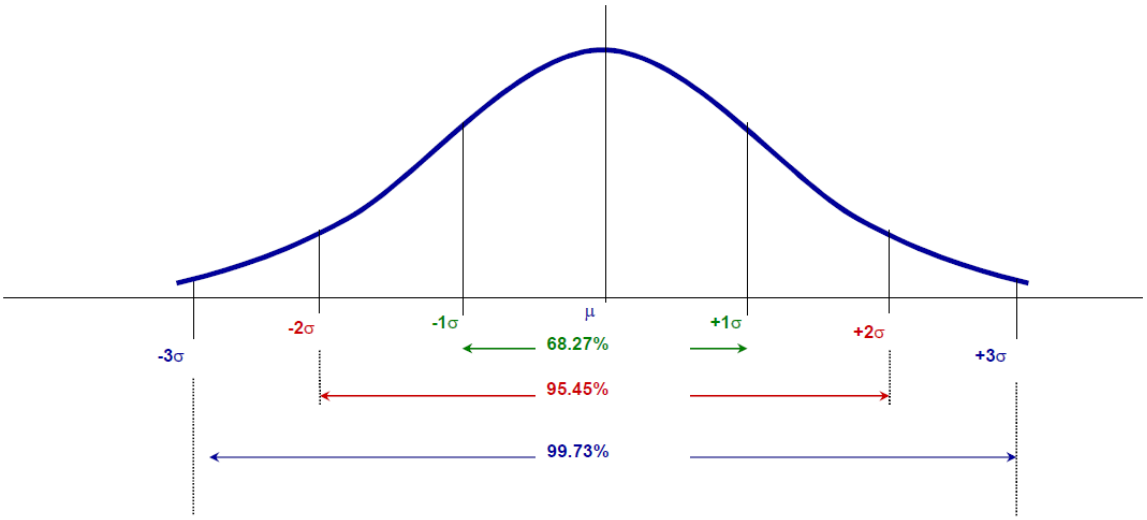
$$P(t_1<t<t_2)= \int_{t1}^{t2} f(t)\,dt$$

Probability distributions



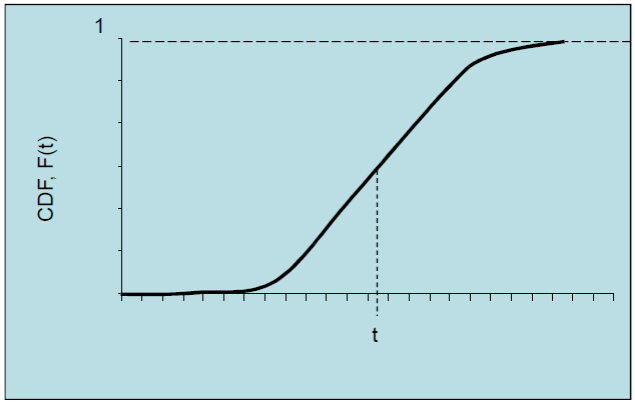
Probability of failure between 500 and 1000 hours is given by the area

Standard Normal distribution



Cumulative distribution function

- The CDF known as F(t)

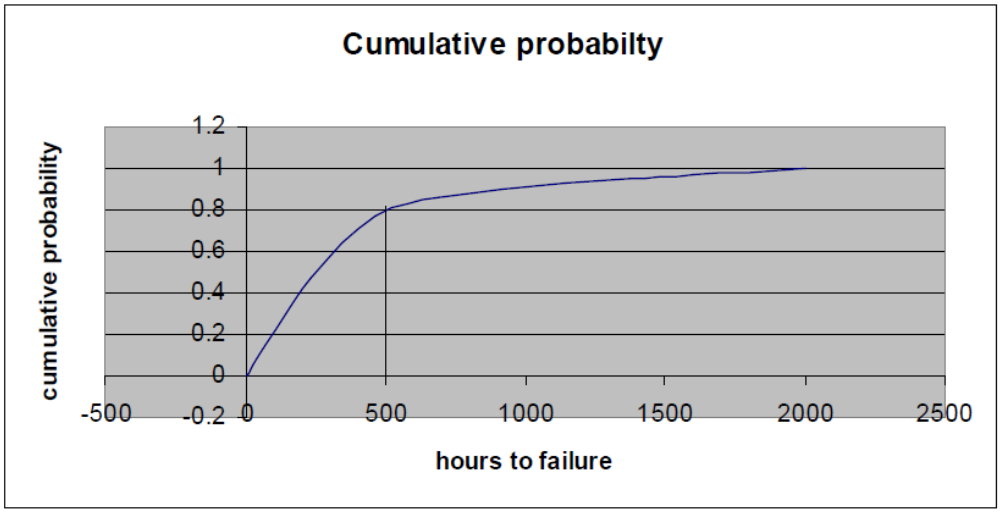


Failure Function, F(t)

$$F(t) = \int_{-\infty}^{t} f(t)\,dt$$

F(t) gives the probability that a measured value will fall between – ∞ and t

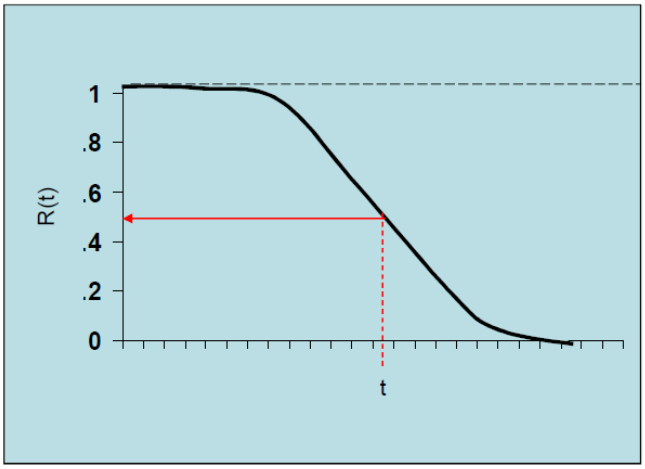
Cumulative distribution



The probability of failure before 500 hours is 0.8 or 80% will have failed by 500hrs

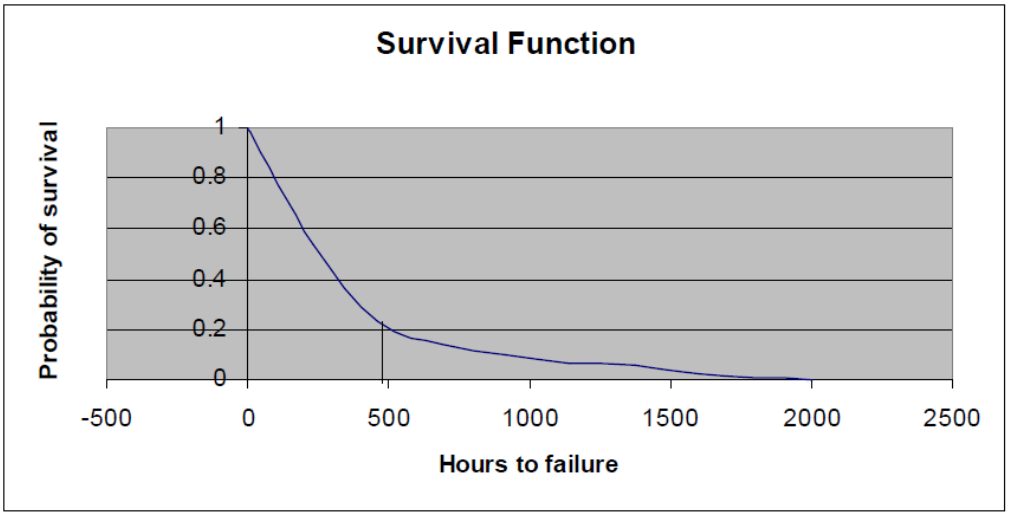
Survival function

- The survival function or reliability function R(t)



R(*t*) = 1 - F(*t*) and
F(*t*) = 1 - R(*t*)

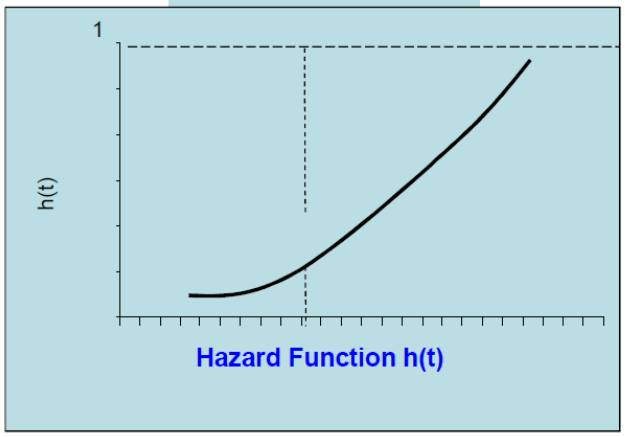
Survival Function



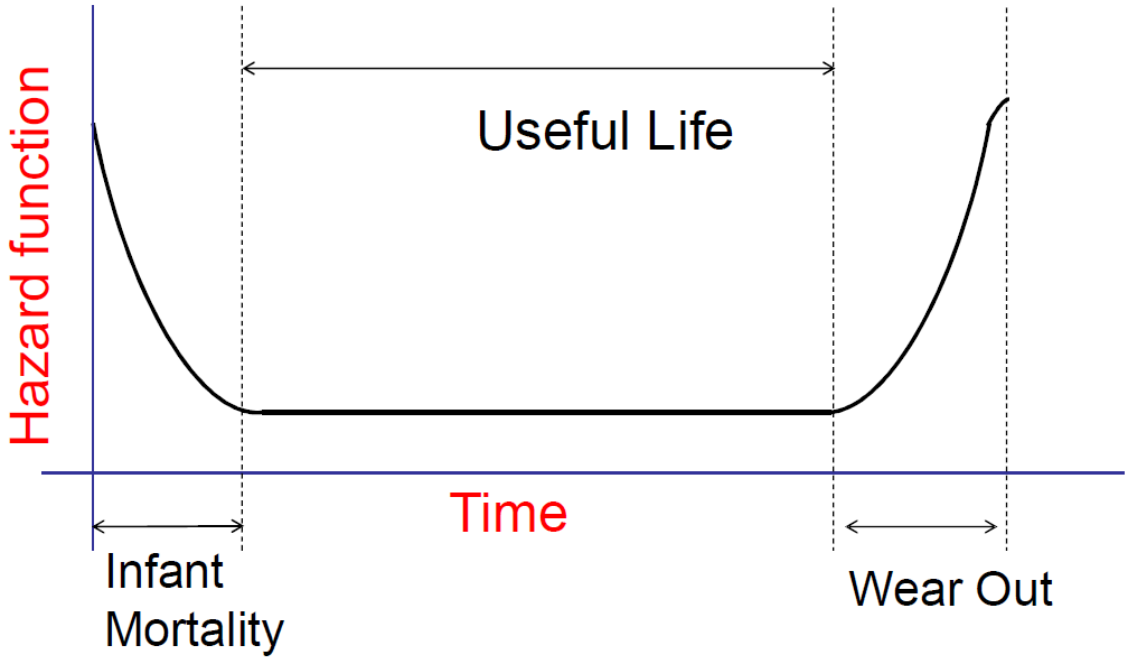
The probability of surviving up to 500 hrs is 0.2
Or 20% have survived up to 500 hrs

Hazard function

- The Hazard function is defined as probability of failure in next time interval given survival to time t
- h(t) = $\frac{f(t)}{1-F(t)} = \frac{f(t)}{R(t)}$
- Figure shows increasing hazard function



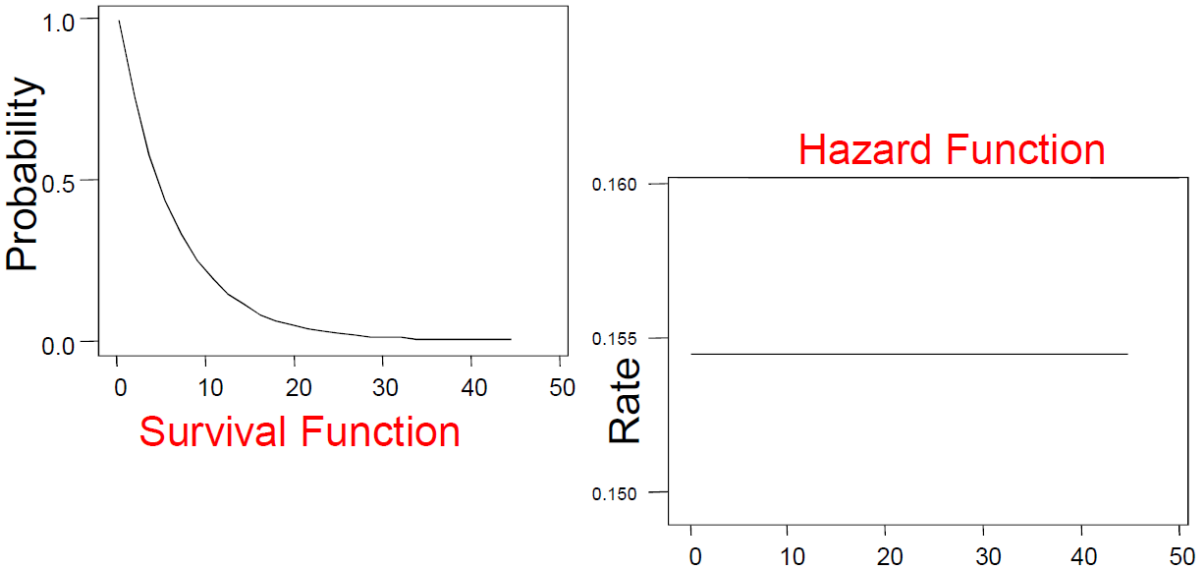
Bath-tub curve



Exponential distribution

- Simplest of all life models
- One parameter, λ
- PDF, $f(t) = \lambda e^{-\lambda t}$
- CDF, $F(t) = 1 - e^{-\lambda t}$ and $R(t) = e^{-\lambda t}$
- Hazard function, $h(t) = \lambda$ i.e. constant
- MTBF = $1/\lambda$ and failure rate = λ
- $1/\lambda$ is the 63rd percentile i.e. time at which 63% of population will have failed

Exponential distribution



Failure rate - example

- 10 components of a particular type in each PCB
- 5 PCBS in each unit
- 200 units in the field
- Total operating time to date for all units is 10,000 hours
- There have been 30 confirmed failures of this component
- The failure rate is given by:
  - 30/5*200*10*10,000 = 0.000003 = 3 fpmh (failures per million hours)
- The MTTF is 1/0.000003 = 333,333

Example

- 100 units in the field
- Total operating hours is 30,000
- Number of confirmed failures is 60
- MTBF = 30,000*100/60 = 50,000
- Removal rate includes all units removed regardless of whether they have failed

REFERENCE AND ACKNOWLEDGEMENT

An Introduction to Reliability and Life Distributions

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