

2 The Ebers-Moll Bipolar Junction Transistor Model

2.1 Introduction

The bipolar junction transistor can be considered essentially as two p-n junctions placed back-to-back, with the base p-type region being common to both diodes. This can be viewed as two diodes having a common third terminal as shown in Fig. 2.1.

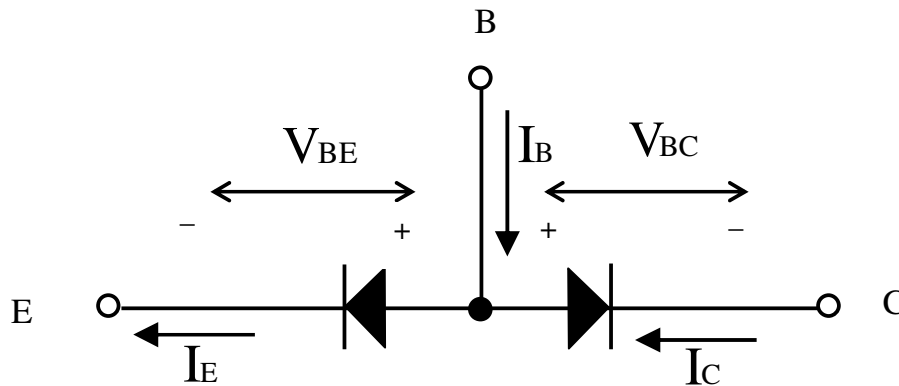
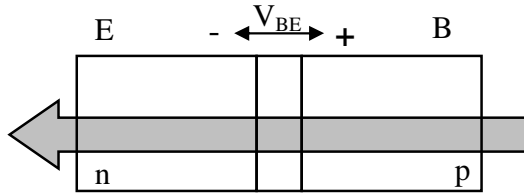


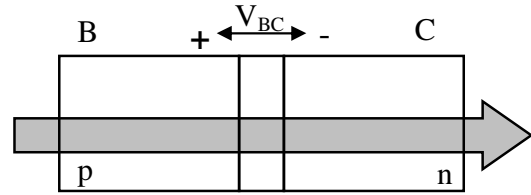
Fig. 2.1 Bipolar Transistor Shown as Two Back-to-Back p-n Junctions

However, the two diodes are not in isolation, but are interdependent. This means that the total current flowing in each diode is influenced by the conditions prevailing in the other. In isolation, the two junctions would be characterized by the normal Diode Equation with a suitable notation used to differentiate between the two junctions as can be seen in Fig. 2.2. When the two junctions are combined, however, to form a transistor, the base region is shared internally by both diodes even though there is an external connection to it. As seen previously, in the forward active mode, α_F of the emitter current reaches the collector. This means that α_F of the diode current passing through the base-emitter junction contributes to the current flowing through the base-collector junction. Typically, α_F has a value of between 0.98 and 0.99. This is shown as the forward component of current as it applies to the normal forward active mode of operation of the device. Note this current is shown as a conventional current in Fig. 2.2. It is equally possible to reverse the biases on the junctions to operate the transistor in the "reverse active mode". In this case, α_R times the collector current will contribute to the emitter current. For the doping ratios normally used the transistor will be much less efficient in the reverse mode and α_R would typically be in the range 0.1 to 0.5.



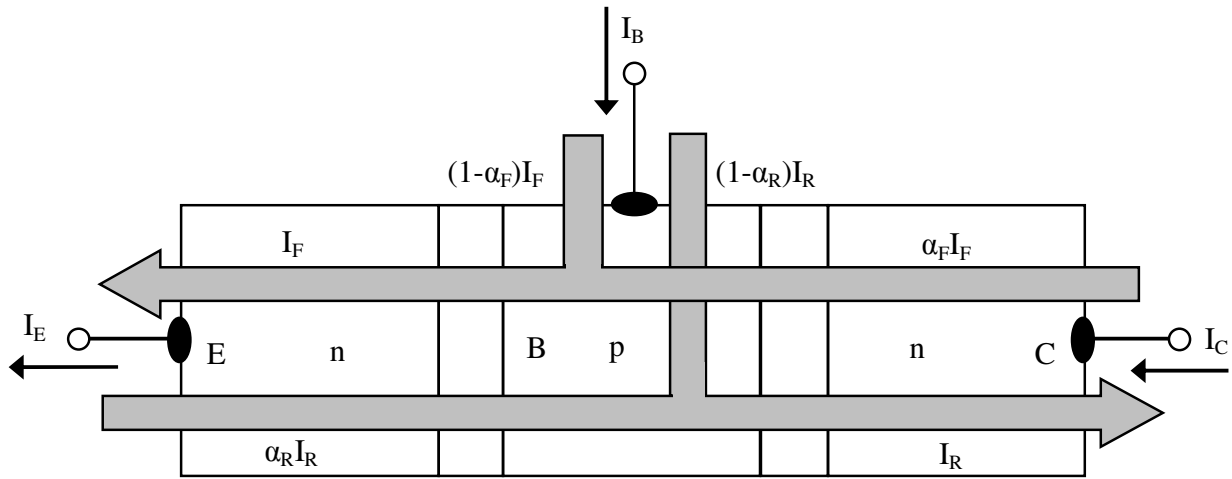
$$I_F = I_{ES}(e^{V_{BE}/V_T} - 1)$$

B - E Junction in Isolation



$$I_R = I_{CS}(e^{V_{BC}/V_T} - 1)$$

B - C Junction in Isolation



Combined B-E and B-C Junctions

Fig. 2.2 The n-p-n Transistor Considered as Combined p-n Junctions

2.2 Ebers-Moll Equations

The Ebers-Moll transistor model is an attempt to create an electrical model of the device as two diodes whose currents are determined by the normal diode law but with additional transfer ratios to quantify the interdependency of the junctions as shown in Fig. 2.3. Two dependent current sources are used to indicate the interaction of the junctions. The interdependency is quantified by the forward and reverse transfer ratios, α_F and α_R . The diode currents are given as:

$$\mathbf{I}_F = \mathbf{I}_{ES} (e^{V_{BE}/V_T} - 1) \quad \text{where} \quad \mathbf{I}_{ES} = qA \left(\frac{D_e p_{e0}}{L_e} + \frac{D_b n_{b0}}{W_b} \right) = qA \left(\frac{D_e n_i^2}{L_e N_e} + \frac{D_b n_i^2}{W_b N_b} \right)$$

$$\mathbf{I}_R = \mathbf{I}_{CS} (e^{V_{BC}/V_T} - 1) \quad \text{where} \quad \mathbf{I}_{CS} = qA \left(\frac{D_c p_{c0}}{L_c} + \frac{D_b n_{b0}}{W_b} \right) = qA \left(\frac{D_c n_i^2}{L_c N_c} + \frac{D_b n_i^2}{W_b N_b} \right)$$

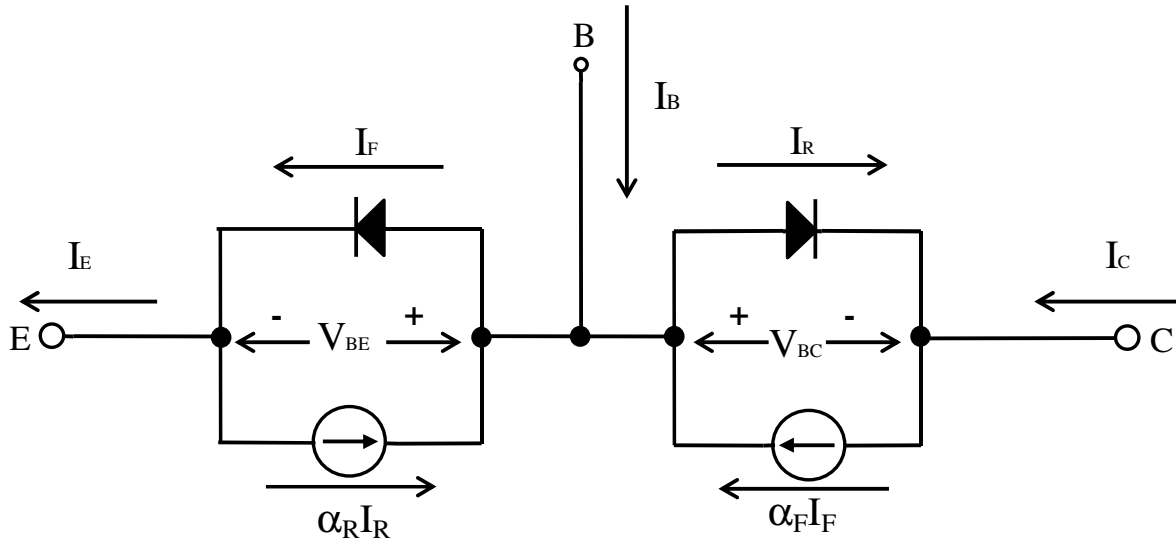
Applying Kirchoff's laws to the model gives the terminal currents as:

$$\begin{aligned} \mathbf{I}_E &= \mathbf{I}_F - \alpha_R \mathbf{I}_R & \alpha_F &= 0.98 - 0.99 \text{ typically} \\ \mathbf{I}_C &= \alpha_F \mathbf{I}_F - \mathbf{I}_R & \alpha_R &= 0.1 - 0.5 \text{ typically} \\ \mathbf{I}_B &= \mathbf{I}_E - \mathbf{I}_C \end{aligned}$$

This gives:

$$\begin{aligned} \mathbf{I}_E &= \mathbf{I}_{ES} (e^{V_{BE}/V_T} - 1) - \alpha_R \mathbf{I}_{CS} (e^{V_{BC}/V_T} - 1) \\ \mathbf{I}_C &= \alpha_F \mathbf{I}_{ES} (e^{V_{BE}/V_T} - 1) - \mathbf{I}_{CS} (e^{V_{BC}/V_T} - 1) \\ \mathbf{I}_B &= (1 - \alpha_F) \mathbf{I}_{ES} (e^{V_{BE}/V_T} - 1) + (1 - \alpha_R) \mathbf{I}_{CS} (e^{V_{BC}/V_T} - 1) \end{aligned}$$

These are called the Ebers-Moll Equations for the bipolar transistor (see Fig. 2.3).



$$\mathbf{I_E = I_{ES} (e^{V_{BE}/V_T} - 1) - \alpha_R I_{CS} (e^{V_{BC}/V_T} - 1)}$$

$$\mathbf{I_C = \alpha_F I_{ES} (e^{V_{BE}/V_T} - 1) - I_{CS} (e^{V_{BC}/V_T} - 1)}$$

$$\mathbf{I_B = (1 - \alpha_F) I_{ES} (e^{V_{BE}/V_T} - 1) + (1 - \alpha_R) I_{CS} (e^{V_{BC}/V_T} - 1)}$$

Fig. 2.3 The Ebers-Moll Model of an n-p-n Bipolar Junction Transistor

2.3 Modes of Operation

The Ebers-Moll BJT Model is a good large-signal, steady-state model of the transistor and allows the state of conduction of the device to be easily determined for different modes of operation of the device. The different modes of operation are determined by the manner in which the junctions are biased. The charge profiles for each mode are shown in Fig. 2.4.

(a) Forward Active Mode

B-E forward-biased, V_{BE} positive

B-C reverse biased, V_{BC} negative

$$e^{V_{BE}/V_T} \gg 1$$

$$e^{V_{BC}/V_T} \ll 1$$

Then from the model,

$$\begin{aligned} I_E &\approx I_{ES} e^{V_{BE}/V_T} && \text{relatively large} \\ I_C &\approx \alpha_F I_{ES} e^{V_{BE}/V_T} = \alpha_F I_E && \text{relatively large} \\ I_B &\approx (1 - \alpha_F) I_{ES} e^{V_{BE}/V_T} = (1 - \alpha_F) I_E && \text{small} \end{aligned}$$

(b) Reverse Active Mode

B-E reverse biased, V_{BE} negative

B-C forward biased, V_{BC} positive

$$e^{V_{BE}/V_T} \ll 1,$$

$$e^{V_{BC}/V_T} \gg 1$$

Essentially the transistor conducts in the opposite direction.

From the model,

$$\begin{aligned} I_E &\approx -\alpha_R I_{CS} e^{V_{BC}/V_T} && \text{moderately high} \\ I_C &\approx -I_{CS} e^{V_{BC}/V_T} && \text{moderate} \\ I_B &\approx (1 - \alpha_R) I_{CS} e^{V_{BC}/V_T} && \text{as high as } 0.5 | I_C | \end{aligned}$$

This mode does not provide useful amplification but is used, mainly, for current steering in switching circuits, e.g. TTL.

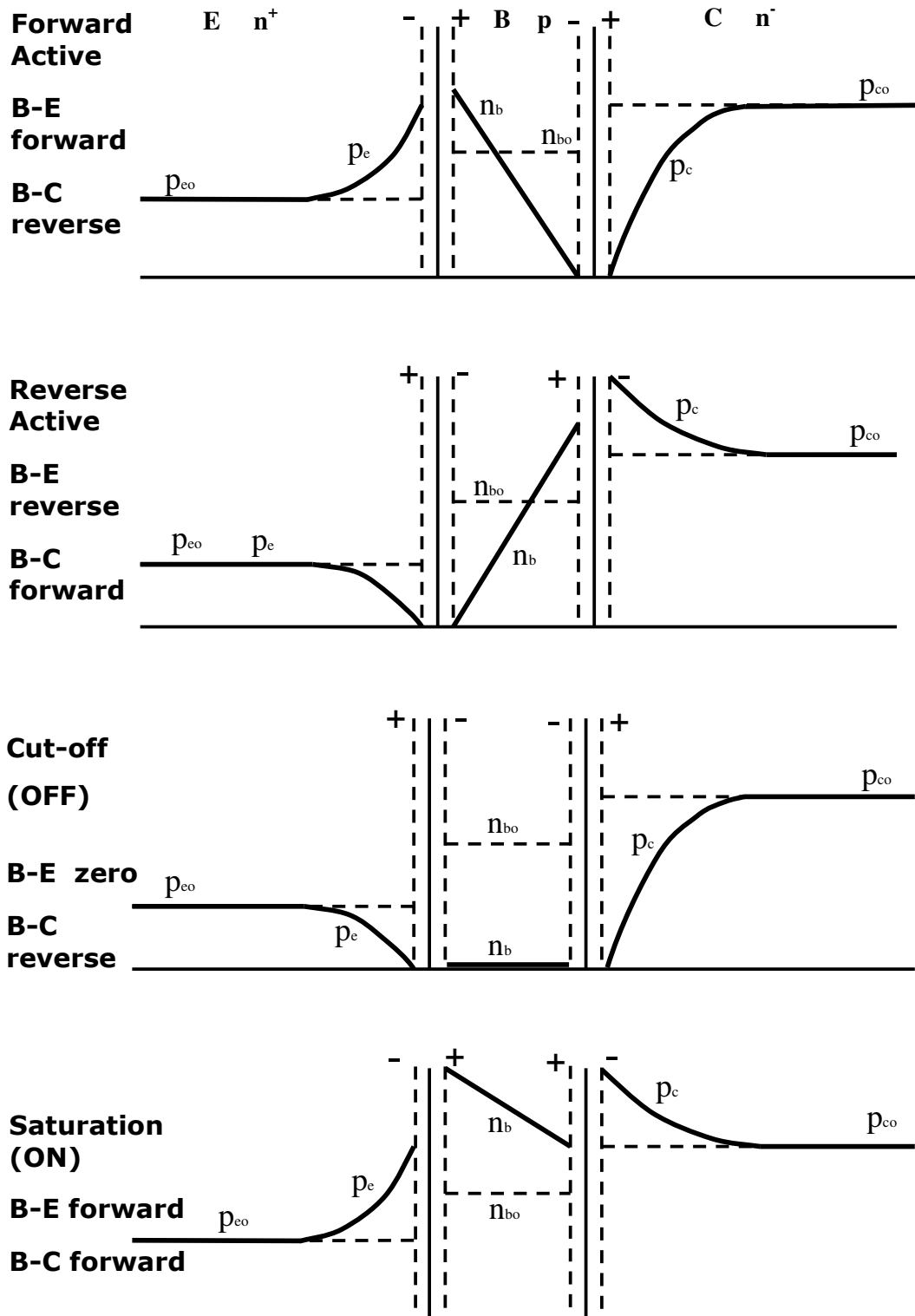


Fig. 2.4 Charge Profiles for Modes of Operation of n-p-n BJT

(c) Cut-Off Mode

B-E unbiased, $V_{BE} = 0V$

B-C reverse biased, V_{BC} negative

$$e^{V_{BE}/V_T} = 1,$$

$$e^{V_{BC}/V_T} \ll 1.$$

Then,

$$I_E \approx \alpha_R I_{CS} \quad \text{leakage current} \quad nA$$

$$I_C \approx I_{CS} \quad \text{leakage current} \quad nA$$

$$I_B \approx -(1 - \alpha_R) I_{CS}$$

This is equivalent to a very low conductance between collector and emitter as shown in Fig. 2.5, i.e. an open switch.

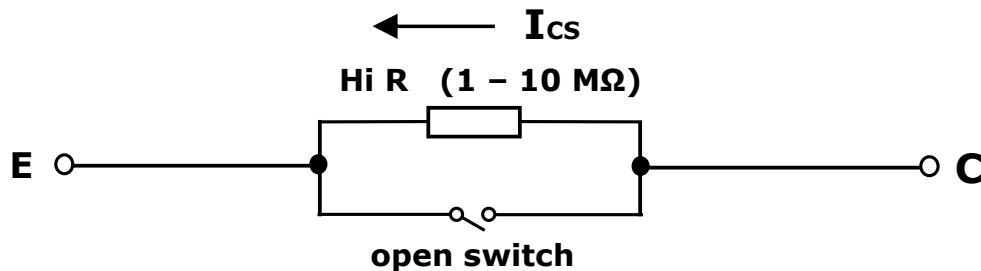


Fig. 2.5 The Cut-Off Mode of Operation as Equivalent to a Leaky Switch

(d) The Saturation Mode

B-E forward biased, V_{BE} positive positive

B-C forward biased, V_{BC} positive

$$e^{V_{BE}/V_T} \gg 1$$

$$e^{V_{BC}/V_T} \gg 1$$

Note: both junctions are forward biased

Then,

$$I_E \approx I_{ES} e^{V_{BE}/V_T} - \alpha_R I_{CS} e^{V_{BC}/V_T}$$

$$I_C \approx \alpha_F I_{ES} e^{V_{BE}/V_T} - I_{CS} e^{V_{BC}/V_T}$$

$$I_B \approx (1 - \alpha_F) I_{ES} e^{V_{BE}/V_T} + (1 - \alpha_R) I_{CS} e^{V_{BC}/V_T}$$

In this case, with both junctions forward biased

$$V_{BE} \approx 0.8V$$

$$V_{BC} \approx 0.7V$$

$$V_{CE} = V_{BE} - V_{BC} = 0.1V$$

There is a 0.1V drop across the transistor from collector to emitter which is quite low while a substantial current flows through the device. In this mode it can be considered as having a very high conductivity and acts as a closed switch with a finite resistance or conductivity.

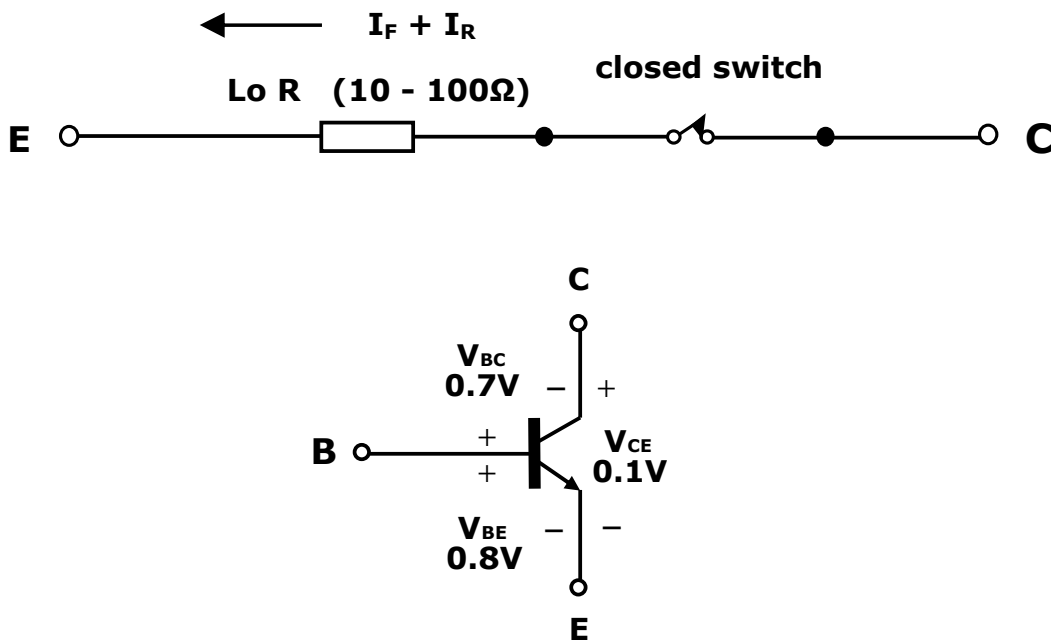


Fig. 2.6 Saturation Mode of Operation Equivalent to a Closed Switch